Math 221 Test 3 **Spring 2009**

Name:______________________________ Score:_________

Instructions: You must show supporting work to receive full and partial credits. No text book, personal notes, formula sheets allowed. **One formula sheet is provided on next page.**

1(**20pts**) Find the general solution of the equations

$$\begin{cases} x' = -7x + 15y \\ y' = -6x + 11y \end{cases}$$

(*Hint:* The characteristic equation is $\lambda^2 - 4\lambda + 13 = 0$.)

2(**20pts**) You are given the fact that $\lambda = 2$ is the only eigenvalue and $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is the only linearly independent eigenvector of the system of equation below. Find the a generalized eigensolution of the system, and then find the general solution of the system.

$$\begin{cases} x' = 4x - 4y \\ y' = x \end{cases}$$

3(**20pts**) Use definition *only* to find the Laplace transform of the function

$$f(t) = \begin{cases} \sin(bt), & t < 1 \\ 0, & t \geq 1 \end{cases}$$

(You can use the formula $\int e^{at}\sin(bt)dt = \frac{a}{a^2+b^2}e^{at}\sin(bt) - \frac{b}{a^2+b^2}e^{at}\cos(bt) + C$.)

4(**20pts**) Find the Laplace transform or inverse transform:

(a) Find $\mathcal{L}\{t^2 \sin t\}(s)$.

(b) If $\mathcal{L}^{-1}\{\frac{7s+10}{s^3+4s^2+5s}\}(t)$.

5(**20pts**) Use the Laplace method to solve the initial value problem

$$x' + 2x = f(t),\ x(0) = 1$$

with $f(t) = \begin{cases} 0, & t < \frac{1}{2} \\ 2t, & t \geq \frac{1}{2}. \end{cases}$

(*Hint:* The solution is $x(t) = e^{-2t} + u(t - \frac{1}{2})(t - \frac{1}{2})$.)

Bonus 2pts: The University of Nebraska system was founded in the year of ________________

The End

Table of Laplace Transforms and Inverse Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}(t)$	$F(s) = \mathcal{L}\{f(t)\}(s)$
$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$, $s > a$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$, $s > a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$, $s > a$
$e^{at} f(t)$	$F(s)\big\rvert_{s\to s-a}$
$u(t-a)f(t)$	$e^{-as}\mathcal{L}\{f(t+a)\}(s)$, alternatively,
$u(t-a)\big[f(t)\big\rvert_{t\to t-a}\big]$	$e^{-as}F(s)$
$\delta(t-a)f(t)$	$f(a)e^{-as}$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
$t^n f(t)$	$(-1)^n \dfrac{d^n}{ds^n}F(s)$, with special case
	$\mathcal{L}^{-1}\{F(s)\}(t) = -\dfrac{1}{t}\mathcal{L}^{-1}\{F'(s)\}(t)$
$\dfrac{f(t)}{t}$	$\displaystyle\int_s^\infty F(\sigma)\, d\sigma$, with special case
	$\mathcal{L}^{-1}\{F(s)\}(t) = t\mathcal{L}^{-1}\left\{\displaystyle\int_s^\infty F(\sigma)\, d\sigma\right\}(t)$
$\displaystyle\int_0^t f(\tau)\, d\tau$	$\dfrac{F(s)}{s}$
$f(t) * g(t)$	$F(s)G(s)$