Math 221 Final Exam **Spring 2009**

Name:________________________ Score:__________

Instructions: You must show supporting work to receive full and partial credits. No text book, personal notes, formula sheets allowed. Turn off all electronic devices except for calculators. **A formula table is provided.**

1(15 pts) Find the general solution to the separable equation $\sin y \dfrac{dy}{dx} = x \cos y + x$. (Either an implicit or explicit solution is acceptable.)

2(20 pts) Solve the initial value problem to the 1st order linear equation $\dfrac{dy}{dx} - \dfrac{2y}{x} = 2x^2, y(1) = 2$. (You must solve it by the method for 1st order linear equations. Any other way will receive a significantly small credit.)

3(20 pts) (a) Use the Euler method to approximate the solution to the initial value problem $y' = t - t^2y^2, y(1) = 0.5$ over the interval $1 \le t \le 2$ at these points: $t = 1, 1.25, 1.5, 1.75, 2$.

(b) Sketch your approximating solution.

4(20 pts) Consider the autonomous equation $\dfrac{dx}{dt} = x^2(2-x)(x-1)$

(a) With the aid of a graphical calculator or by hand, sketch an equation plot and a phase line in the interval $[-1, 3]$.

(b) Classify the stability of all equilibrium solutions in the interval.

(c) Sketch a solution portrait of the equation, and state the limit of $\lim_{t\to\infty} x(t)$ for the solution with the initial condition $x(0) = 0.5$.

5(15 pts) The characteristic equation for a homogeneous, 3rd order linear, and constant coefficient equation $ay''' + by'' + cy' + dy = 0$ is factored as $ar^3 + br^2 + cr + d = (r+2)(r^2 + 2r + 5) = 0$. Find the general solution to the differential equation.

6(20 pts) Use the method of undetermined coefficients to find a a particular solution $y_p(x)$ to the equation $y' - 2y = 2e^{2x}$. (No credit for any other method!)

7(20 pts) Use the method of variation of parameters to find a particular solution to the equation $xy'' + b(x)y' + c(x)y = 2$ given that $y_1(x) = 1, y_2(x) = \sqrt{x}$ are two solutions to the homogeneous equation. (Solution by any other method receives no points.)

8(20 pts) Find the solution to the equations

$$\begin{cases} x' = 7x - 6y \\ y' = 9x - 8y \end{cases}$$

satisfying the initial conditions: $x(0) = 1, y(0) = 2$.

9(20 pts) Find the following Inverse Laplace Transforms:

(a) $\mathcal{L}^{-1}\left\{\dfrac{1}{2s^2 + s}\right\}(t)$.

(b) $\mathcal{L}^{-1}\left\{\dfrac{2s}{s^2 + 8s + 20}\right\}(t)$.

Formula Table

$e^{at}f(t)$	$F(s)\big\|_{s\to s-a}$
$u(t-a)f(t)$	$e^{-as}\mathcal{L}\{f(t+a)\}(s)$
$u(t-a)\left[f(t)\big\|_{t\to t-a}\right]$	$e^{-as}F(s)$
$(f * g)(t)$	$F(s)G(s)$

10(10 pts) Use the definition for Laplace Transform to find $\mathcal{L}\{2\}(s)$. (Any other method receives no point!)

11(20 pts) Use the method of Laplace Transform to solve the initial value problem:

$$x'(t) + 6x(t) = e^{3t}u(t-2), \quad x(0) = 3.$$

END