**Math 221-02 Test 1** **Spring 2008**

Name:_____ Score:_____

**Instructions:** You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(15pts) Use Euler's method with step size $h = 0.5$ to approximate the solution to the IVP: $y' = -xy + 1,\ y(2) = 3$ in the interval $[2, 4]$. Sketch your approximating solution.

2(20pts) Consider the autonomous equation $\dfrac{dx}{dt} = x(4 - x^2)$

(a) With the aid of a graphical calculator or by hand, sketch an equation plot and a phase line in the interval $[-3, 3]$.

(b) Classify the stability of all equilibrium solutions in the interval.

(c) Sketch a solution portrait of the equation, and state the limit of $\lim_{t\to\infty} x(t)$ for the solution with the initial condition $x(0) = 1$.

3(15pts) Find a general solution to the linear equation $(x^2 + 1)y' + 3xy = 6x$.

4(15pts) Find the solution to the IVP of a separable equation $x\dfrac{dy}{dx} - y^2 = 2x^2y^2,\ y(1) = 1$.

5(10pts) Use partial fraction to find the explicit, general solution to the equation $\dfrac{dx}{dt} = 1 - x^2$.

6(20pts) Initially a room of $12 \times 12 \times 10 = 1,440$ ft$^3$ is high on carbon monoxide (CO) concentration at 50 parts-per-million (ppm), i.e. 0.005%. Fresh air (with zero CO trace) is pumped into the room to dilute the CO gas at a rate of 2 ft$^3$/s, and the well-mixed air leaves the room at the same rate.

(a) Derive a differential equation with initial values for the amount of CO in volume at any time.

(b) How long will it take for the CO level to drop below 25 ppm?

**END**