Math 221-02 Final Exam **Spring 2008**

Name:________________________ Score:________

Instructions: You must show supporting work to receive full and partial credits. No text book, personal notes, formula sheets allowed. Turn off all electronic devices except for calculators. **A formula table is provided.**

1(15 pts) Find the general solution to the separable equation $2y\dfrac{dy}{dx} = xy^2 + x$. (Either an implicit or explicit solution is acceptable.)

2(20 pts) Solve the initial value problem to the 1st order linear equation $y' + 2xy = 2x, y(0) = 2$. (You must solve it by the method for 1st order linear equations. Any other way will receive significantly small credit.)

3(20 pts) (a) Use the Euler method to approximate the solution to the initial value problem $x' = 4t^2 - tx^2, x(1) = 2$ over the interval $1 \le t \le 2$ at these points: $t = 1, 1.25, 1.5, 1.75, 2$.

(b) Sketch your approximating solution.

4(20 pts) Consider the autonomous equation $\dfrac{dx}{dt} = x^2(2 - x)$

(a) With the aid of a graphical calculator or by hand, sketch an equation plot and a phase line in the interval $[-1, 3]$.

(b) Classify the stability of all equilibrium solutions in the interval.

(c) Sketch a solution portrait of the equation, and state the limit of $\lim_{t\to\infty} x(t)$ for the solution with the initial condition $x(1) = 3$.

5(15 pts) The characteristic equation for a homogeneous, 4rd order linear, and constant coefficient equation $a_0y^{(4)} + a_1y''' + a_2y'' + a_3y' + a_4y = 0$ is factored as $a_0r^4 + a_1r^3 + a_2r^2 + a_3r + a_4 = 2(r+1)(r+1)(r^2+4r+5) = 0$. Find the general solution to the differential equation.

6(20 pts) Use the method of undetermined coefficients to find a a particular solution $y_p(x)$ to the equation $y''+2y' = 2$. (No credit for any other method!)

7(20 pts) Use the method of variation of parameters to find a particular solution to the equation $xy''+b(x)y'+c(x)y = 2$ given that $y_1(x) = 1, y_2(x) = \ln x$ are two solutions to the homogeneous equation. (You can use the formula $\int \ln x dx = x \ln x - x + C$.)

8(20 pts) Find the solution to the equations

$$\begin{cases} x' = -10x + 12y \\ y' = -9x + 11y \end{cases}$$

satisfying the initial conditions: $x(0) = 2, y(0) = 1$.

9(20 pts) Find the following Inverse Laplace Transforms:

(a) $\mathcal{L}^{-1}\left\{\dfrac{e^{-3s}}{3s^2 + s}\right\}(t)$.

(b) $\mathcal{L}^{-1}\left\{\dfrac{3s}{s^2 + 6s + 10}\right\}(t)$.

Formula Table

$e^{at}f(t)$	$F(s)\big\|_{s\to s-a}$
$u(t-a)f(t)$	$e^{-as}\mathcal{L}\{f(t+a)\}(s)$
$u(t-a)\left[f(t)\big\|_{t\to t-a}\right]$	$e^{-as}F(s)$
$\delta(t-a)f(t)$	$f(a)e^{-as}$

10(10 pts) Use the definition for Laplace Transform to find $\mathcal{L}\{e^{2t}\}(s)$. (Any other method receives no point!)

11(20 pts) Use the method of Laplace Transform to solve the initial value problem:

$$x'(t) + 3x(t) = t^2\delta(t-3), \quad x(0) = 2.$$

END