Math 221 Test 3 **Fall 2015**

Print Your Name:_____ Score:_____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(10pts) Consider the system of equations $\mathbf{x}' = A\mathbf{x}$ with $A = \begin{bmatrix} -7 & 10 \\ -4 & 5 \end{bmatrix}$.

(a) Find a general solution of the system of equations. You must show work to justify your answer.

(b) Sketch a phase portrait for the system of equations.

2(15pts) Consider the system of equations

$$\begin{cases} x' = 2x - y \\ y' = x^2 - y \end{cases}$$

(a) Find all equilibrium solutions to the system.

(b) Use linearization to classify each equilibrium point as unstable saddle, unstable (spiral) source, or stable (spiral) sink.

3(**10pts**) Use definition *only* to find the Laplace transform of the function

$$f(t) = \begin{cases} 2, & t < 3 \\ 0, & t \geq 3 \end{cases}$$

Using any other method receives no credit.

4(**20pts**) Find the Laplace transforms $\mathcal{L}\{f(t)\}(s)$ for the following functions

(a) $f(t) = (t+1)^2$

(b) $f(t) = u(t-2)g(t)$ if $\mathcal{L}\{g(t+2)\}(s) = \dfrac{1}{\sqrt{s^2+4}}$.

5(**20pts**) Find the inverse Laplace transforms $\mathcal{L}^{-1}\{F(s)\}(t)$ for the following functions

(a) $F(s) = \dfrac{2s}{s^2+2s+5}$

(b) $F(s) = \dfrac{s+3}{s^2+3s+2}$

6(**15pts**) Use the Laplace transformation method to solve the initial value problem

$$x' - x = e^{-t^2}\delta(t-1), \quad x(0) = 0.$$

7(**10pts**) Consider the differential equation for a mass-spring system with piece-wise forcing

$$x'' + 2x' + 3x = f(t), \quad x(0) = 0,\ x'(0) = 1$$

with

$$f(t) = \begin{cases} 2t, & 0 \leq t < 1 \\ 0, & 1 \leq t < 3 \\ 1, & 3 \leq t < \infty. \end{cases}$$

Find *only* $X(s) = \mathcal{L}\{x(t)\}(s)$. Do not simplify $X(s)$. Do not solve for $x(t)$.

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2 pt Bonus Question: The Nebraska State River is _____.

END