

Name: _____

Score: _____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(15pts) Find the general solution of the equation $2y^{(4)}(x) - y'''(x) - y''(x) = 0$, where $y^{(4)}(x)$ denotes the 4th derivative of $y(x)$.

2(15pts) Use the method of reduction of order (i.e. variation of parameter) to find a second solution to the equation $x^2y'' - xy' + y = 0$ if one solution is given: $y_1(x) = x$.

- 3(15pts) Use the method of Undetermined Coefficient to find the **FORM** only for a particular solution to the equation $y' - 2y = 2x + \sin(2x) + e^{2x}$. Do not solve for the undetermined coefficients.

- 4(20pts) Use the method of Variation of Parameters to find a particular solution to the equation

$$(x^2 + 2x)y'' - 2(x + 1)y' + 2y = 2(x^2 + 2x)^2$$

for which two linearly independent solutions to the homogeneous equation are given: $y_1(x) = x + 1, y_2(x) = x^2$.

5(15pts) Verify that $\vec{v} = \begin{bmatrix} 3-i \\ 2 \end{bmatrix}$ is an eigenvector of $A = \begin{bmatrix} -5 & 10 \\ -4 & 7 \end{bmatrix}$ and the corresponding eigenvalue is $\lambda = 1 + 2i$.

6(20pts) Consider $A = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$.

(a) Find all eigenvalues of A .

(b) Find all eigenvectors of A .

(c) Find a general solution to the system of equations $\vec{x}'(t) = A\vec{x}(t)$.