Math 221 Final Exam **Fall 2013**

Name:______________________________ Score:__________

Instructions: You must show supporting work to receive full and partial credits. No text book, personal notes, formula sheets allowed. Turn off all electronic devices except for calculators.

1(15 pts) (a) Use the Euler method to approximate the solution to the initial value problem $y' = t - y,\ y(1) = 0.1$ in the interval $1 \le t \le 2$ with a step-size $h = 0.25$.

(b) Sketch your approximating solution.

2(20 pts) Solve the initial value problem $\dfrac{dy}{dx} - 2xy = 2x^3, y(0) = 1$.

3(20 pts) Determine the type of the equation and find the general solution to the equations. (Either an implicit or explicit solution is acceptable.)

(a) $(yx + y)dx + xdy = 0$.

(b) $(3x + 2xy)dx + (x^2 + y)dy = 0$.

4(20 pts) Consider the autonomous equation $\dfrac{dx}{dt} = x(x-1)^2(2-x)$

(a) Sketch the phase line of the equation in the interval $[-1, 3]$.

(b) Classify the stability of all equilibrium solutions in the interval.

(c) Sketch a solution portrait of the equation.

(d) State the limit of $\lim_{t\to\infty} x(t)$ for the solution with the initial condition $x(-1) = 1.5$.

5(15 pts) The characteristic equation for a homogeneous linear equation of constant coefficients, $a_5y^{(5)} + a_4y^{(4)} + a_3y''' + a_2y'' + a_1y' + a_0y = 0$, is factored as $a_5r^5 + a_4r^4 + a_3r^3 + a_2r^2 + a_1r + a_0 = (r^2 + r + 1)^2(r + 1) = 0$. Find the general solution to the differential equation.

6(20 pts) Use the method of undetermined coefficients to find the form of a particular solution $y_p(x)$ to the equation $y'' - y' - 2y = 2e^{2x} + x$. You do not need to find the values of the coefficients.

7(20 pts) (a) Verify that $y_1(x) = 1, y_2(x) = \sqrt{x}$ are two linearly independent solutions to the homogeneous equation $2xy'' + y' = 0$.

(b) Use the method of variation of parameters to find a particular solution to the nonhomogeneous equation $2xy'' + y' = 3$.

8(20 pts) Use the method of eigenvalues to find the solution to the initial value problem

$$\begin{cases} x' = -8x + 9y \\ y' = -6x + 7y \end{cases}, \quad x(0) = 2, y(0) = 1.$$

9(20 pts) Find the following Inverse Laplace Transforms:

(a) $\mathcal{L}^{-1}[e^{-2s}F(s-2)](t)$ if $\mathcal{L}^{-1}[F(s)] = \sqrt{t}$.

(b) $\mathcal{L}^{-1}\left[\dfrac{2s}{s^2 + 8s + 20}\right](t)$.

Formula Table

$\mathcal{L}[e^{at}f(t)](s) = \mathcal{L}[f(t)](s-a)$
$\mathcal{L}[t^n f(t)](s) = (-1)^n \dfrac{d}{ds}\mathcal{L}[f(t)](s)$
$\mathcal{L}[u(t-a)f(t-a)](s) = e^{-as}\mathcal{L}[f(t)](s)$
$\mathcal{L}[(f * g)(t)](s) = \mathcal{L}[f(t)](s) \cdot \mathcal{L}[g(t)](s)$

10(10 pts) Use the definition to find the Laplace Transform of the function $f(t) = e^t, t < 2,\ f(t) = 0, t \ge 2$. (Any other method receives no point!)

11(20 pts) Use the method of Laplace Transform to solve the initial value problem:

$$x'' + 2x' + x = t, \quad x(0) = -1, x'(0) = 0.$$

END