Math 221 Final Exam **Fall 2011**

Name:______________________________ Score:_________

Instructions: You must show supporting work to receive full and partial credits. No text book, personal notes, formula sheets allowed. One formula sheet will be provided.

1(**20pts**) (a) Sketch the phase line of equation $y' = (y^2 - 9)y^2$ and classify the stability of the equilibrium points.

(b) If $y(0.2) = 1$, what is $y(0.21)$ according to Euler's method for the equation of (a) above?

2(**15pts**) Find the solution to the initial value problem $y\frac{dy}{dx} = xy^2 + x,\ y(1) = 0$.

3(**15pts**) Find a general solution to the linear equation $x\frac{dy}{dx} + 2y = x$.

4(**20pts**) (a) Find a general solution to the equation $y''' + y'' + y' = 0$.

(b) Find a particular solution to the nonhomogeneous equation $y''' + y'' + y' = 2x$ and then a general solution.

5(**20pts**) (a) Verify that $y_1(x) = \frac{1}{x}, y_2(x) = x$ are solutions to the homogeneous equation $x^2y'' + xy' - y = 0$.

(b) Use the method of variation of parameters to find a particular solution of $x^2y'' + xy' - y = x$.

6(**20pts**) Use the method of phase plane to sketch a phase portrait of the system of equations $\begin{cases} x' = x - 2 \\ y' = 2x - y. \end{cases}$ Make sure to include all essential features (e.g. both x- and y-nullclines, equilibrium points, vector fields both on and off the nullclines, special solutions that approach the equilibrium points in forward or backward time). Classify the equilibrium points as sink, source, saddle, or spiral sink, source, center.

7(**25pts**) Consider the system of equations $\begin{cases} x' = 7x - 3y \\ y' = 18x - 8y \end{cases}$

(a) Find the eigenvalues of the system.

(b) Find a general solution of the system.

(c) Find the solution of the system with the initial condition $x(0) = 2,\ y(0) = 5$.

8(**20pts**) (a) Use definition to find the Laplace transform of $f(t) = \begin{cases} 0, & t < a \\ 1, & t \geq a \end{cases}$

(b) Use the **Convolution Formula** to find the inverse Laplace transform of $\dfrac{1}{s(s^2+1)}$.

9(**25pts**) Use the Laplace Transform method to solve the system of equations $\begin{cases} x' = -y + \delta(t-3) \\ y' = x \\ x(0) = 0,\ y(0) = 1. \end{cases}$

10(**20pts**) (a) Find all equilibrium points of the system of equations $\begin{cases} x' = 4x - y^2 \\ y' = y - \frac{2}{x} \end{cases}$

(b) Use linearization to determine the stability of the equilibrium points.

Bonus 2pts: How many Martian days are in one Martian year? ______

The End