Name:______________________ Score:______________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(15pts) Solve the initial value problem $y'' - 3y' + 2y = 0, y(0) = 1, y'(0) = 2$.

2(20pts) The linear differential equation of constant coefficients $a_5y^{(5)} + a_4y^{(4)} + a_3y^{(3)} + a_2y'' + a_1y' + a_0y = 1 + 2x + 3\sin(2x)$ has $r = 0, 0, 1, \pm 2i$ as the roots of its characteristic equation.

(a) Find a general solution to the homogeneous equation.

(b) Find a **FORM** for a particular solution to the nonhomogeneous equation. Justify your answer.

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3(20pts) One solution $y_1(x) = x^2$ is given for the differential equation $x^2y'' - 2xy' + 2y = 0$. Use the method of reduction of order to find a second, linearly independent solution.

4(20pts) Use the method of variation of parameters to find a particular solution to the equation $x^2y'' + xy' - y = 2x^2$ given that $y_1(x) = \frac{1}{x}, y_2(x) = x$ are solutions to the homogeneous equation.

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5(25pts) For this system of linear equations

$$\begin{cases} x_1' = 2x_1 - 7x_2 + 3x_3 \\ x_2' = \quad -5x_2 + 3x_3 \\ x_3' = \quad -15x_2 + 7x_3 \end{cases}$$

(a) For the coefficient matrix A of the system, find the characteristic polynomial and the eigenvalues.

(b) Find an eigenvector for each real-valued eigenvalue.

(c) You are given the fact that if A has a pair of complex eigenvalues then an eigenvector for one of the eigenvalues is $\begin{bmatrix} 1 \\ 1 \\ 2+i \end{bmatrix}$. Verify this in fact is the case.

(d) Find a real-valued general solution to the system of the differential equations.

(... *The End*)