Math 221 Test 1 **Spring 2011**

Name:____________________________ Score:____________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(15pts) Use Euler's method to approximate $y(1)$ at $x = 1$ to the third decimal place where $y(t)$ is the solution to the IVP: $y' = 2x - y^2,\ y(0.5) = 0$ using a step size 0.25. Sketch your approximating solution.

2(20pts) Consider the autonomous equation $\dfrac{dx}{dt} = x^2(2 - x)$.

(a) With the aid of a graphical calculator or by hand, sketch the phase line in the interval $[-3, 3]$.

(b) Classify the stability of each equilibrium point in the interval.

(c) Sketch a solution portrait of the equation, including the one with the initial condition $x(1) = 1$. What is the limit of $\lim_{t\to\infty} x(t)$ of this particular solution?

3(15pts) Find a general solution to the linear equation $(x^2 - 1)y' - 2xy = 4x$ for $x > 1$.

4(15pts) Determine the type of the equation $x\dfrac{dy}{dx} = (x + 1)\sqrt{y + 1}$, and then find the solution with the initial condition $y(1) = 0$.

5(10pts) Use an appropriate substitution to transform this equation $x^2\dfrac{dy}{dx} = 2x^2 + y^2$ to a separable equation in the new variable. Derive the equation in the new variable but **DO NOT SOLVE THE EQUATIONS.**

6(10pts) Initially a room of $12 \times 12 \times 10 = 1,440$ ft^3 is high on carbon monoxide (CO) concentration at 50 parts-per-million (ppm), i.e. 0.005%. Fresh air (with zero CO trace) is pumped into the room to dilute the CO gas at a rate of 2 ft^3/s, and the well-mixed air is pumped out at the same rate. Derive a differential equation with initial conditions for the amount of CO in volume at any time. **DO NOT SOLVE THE EQUATIONS.**

7(10pts) The per-capita birth rate of a deer population $P(t)$ is assumed to be a constant but the per-capita death rate is proportional to the square root of the population. If $P(0) = 1000$, derive an initial value problem for the deer population. **DO NOT SOLVE THE EQUATIONS.**

END