Math 221 Test 3 **Fall 2004**

Name:______________________________ Score:__________

Instructions: You must show supporting work to receive full and partial credits. No text book, personal notes, formula sheets allowed. One formula sheet will be provided.

1(**15pts**) Find the solution to the linear equation with initial value: $y'(t) - \frac{2t}{1+t^2}y(t) = 1,\ y(0) = 0.$

2(**15pts**) Use the method of variation of parameters to find a particular solution to the equation $(1+\tan x)y'' - 2y' + (1-\tan x)y = (1+\tan x)^2, x > 0$, given that 2 linearly independent solutions to the homogeneous equation are given as $y_1(x) = \cos x,\ y_2(x) = e^x$. (Usable identity: $\int \sec x dx = \ln|\tan x + \sec x| + C$)

3(**15pts**) (a) Find all equilibrium solutions of the predator-prey system: $\frac{dx}{dt} = x(2-x-y),\ \frac{dy}{dt} = y(x-1)$.

(b) For the equilibrium solution not on either of the axes, find the linearized system of equations and determine the local stability of the equilibrium solution.

4(**15pts**) Find a general solution to the system of equations, $\frac{dx}{dt} = -7x + 4y,\ \frac{dy}{dt} = -8x + 5y$, and sketch a phase portrait of the system.

5(**15pts**) If a system of linear equations of real coefficients, $\mathbf{x}' = A\mathbf{x}$, has an eigenvalue $-1+3i$ and a corresponding eigenvector $\vec{\xi} = \begin{bmatrix} 2 \\ 1+2i \end{bmatrix}$, find the general solution of the system, and sketch a possible phase portrait of the system.

6(**15pts**) If a system of linear equations of real coefficients, $\mathbf{x}' = A\mathbf{x}$, has eigenvalues $0, -2$ and corresponding eigenvectors $\vec{\xi}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\vec{\xi}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ respectively, find the solution with the initial condition $x(0) = 0, y(0) = 1$, and sketch a possible phase portrait of the system.

7(**10pts**) It is given that the system of equations $\frac{dx}{dt} = y,\ \frac{dy}{dt} = -4x + 4y$ has a double eigenvalue 2, and only one linearly independent eigenvector $\vec{\xi} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Find the general solution to the system.

Bonus 3pts: Fill in the blank: If you can dodge a wrench you can ____________________.

The End