Math 221 Test 1 **Fall 2004**

Name:_____ Score:_____

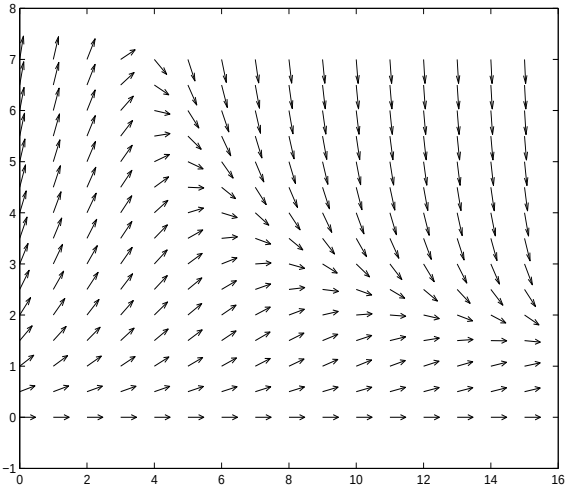
Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(15pts) (a) Verify whether or not $y(x) = x^3 - 2$ is a solution to the equation: $x^2y'' - 6y = 12$.

(b) Find all constants of r so that $y(x) = x^r$ is a solution to the equation: $2x^2y'' + 5xy' - 2y = 0$.

2(10pts) (a) The slope field of a differential equation $\frac{dy}{dt} = f(t, y)$ is given in figure below. Sketch the solution through these points: $y(0) = 1,\ y(1) = 0$.

(b) Sketch the zero isocline of the equation $\frac{dy}{dt} = (y - 1)(1 - ty)$ in the right space below, and sketch the vector field on the zero isocline. Must the zero isocline be solution curves to the equation? Explain your answer.



3(15pts) Use Euler's method to approximate the solution of the IVP: $\dfrac{dy}{dt} = t - 2y^2,\ y(1) = 2$ at these points: $y(1.25), y(1.5), y(1.75), y(2)$ Sketch your approximating solution.

4(15pts) Find the solution to the initial value problem $(1 + t^2)\dfrac{dx}{dt} = t + tx^2,\ x(0) = 1$.

5(15pts) Find the solution to the equation $\dfrac{d^2y}{dt^2} = -t + 1,\ \dfrac{dy}{dt}(1) = 1,\ y(1) = 2$.

6(10pts) A circular water spot 10cm in diameter and 5mm in thickness is drying up. The rate at which the volume decreases is proportional to 3/2 power of the top surface area and assume the thickness stays the same throughout the process. Derive a differential equation to model the situation. If the spot is half of its initial size in area 30 minutes later, what is the size in percentage of the initial 1 hours later?

7(10pts) The burner was turned off from a kettle of boiling water at 100°C. 10 minutes later, the water was 80°C in a room of 30°C in temperature. When would the water reach 50°C?

8(10pts) Let x be the population of a prey and y be the population of a predator. Assume the per-capita growth rate of x and the per-capita growth rate of y are both linear functions of x and y. Assume also the logistic growth for each species in the absence of the other. Write down a system of differential equations for x, y. State the sign of each system parameter.

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