Nov. 6, 2003 **Math 221 Test 2** **Fall 2003**

Name:_____ Score:_____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(18pts) (a) Find the solution to the initial value problem $x'' + 3x' + 2x = 0, x(0) = 0, x'(0) = 1$.

(b) Find a general solution to the equation $x''' + 4x'' + 5x' + 2x = 0$.

2(16pts) (a) Use the method of undetermined coefficient to find a particular solution to the equation $x'' + 4x = 4\cos(2t)$

(b) If the roots of the auxiliary equation of the homogeneous equation $ax''' + bx'' + cx' + dx = 0$ are $-2, -1 \pm 3i$, find the FORM of a particular solution to the nonhomogeneous equation $ax''' + bx'' + cx' + dx = t + te^{-2t} + e^{-t}\cos(3t)$.

3(16pts) Use the method of variation of parameter to find a particular solution to the equation $tx'' - (t+1)x' + x = t^2$ for which two linearly independent solutions to the homogeneous equation are given as $x_1(t) = e^t, x_2(t) = t + 1$. (You need to use integration by parts to find the integral $\int te^{-t}dt$ somewhere in your work.)

4(15pts) Find a general solution to the Cauchy-Euler equation $t^2x'' + 2tx' - 6x = 0$.

5(15pts) A 2-kg mass is attached to a spring hanging from the ceiling, thereby causing the spring to stretch 20 cm upon coming to rest at equilibrium. At time $t = 0$, the mass is displaced 5 cm below the equilibrium and released. At this same instant, an external force $F(t) = 0.3\cos t$ N is allied to the system. If the damping constant for the system is 5 N-sec/m, determine the equation of motion for the mass. Find the steady-state solution for the system.

6(10pts) A mass-spring system with damping consists of a 7-kg mass, a spring with spring constant 3 N/m, a frictional component with damping constant 3 N-sec/m, and an external force $f(t) = 10\cos(10t)$ N. Using a 5-Ω resister, construct an RLC series circuit that is the analog of this mechanical system in the sense that the two systems are governed by the same differential equation.

7(10pts) An LC series circuit has a voltage source given by $E(t) = 30\sin(50t)$ V, an inductor of 2 H, and a capacitor of 0.02 F (but no resister). What is the current in this circuit for $t > 0$ if at $t = 0, I(0) = q(0) = 0$?

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