Oct. 2, 2003 **Math 221 Test 1** **Fall 2003**

Name:_____ Score:_____

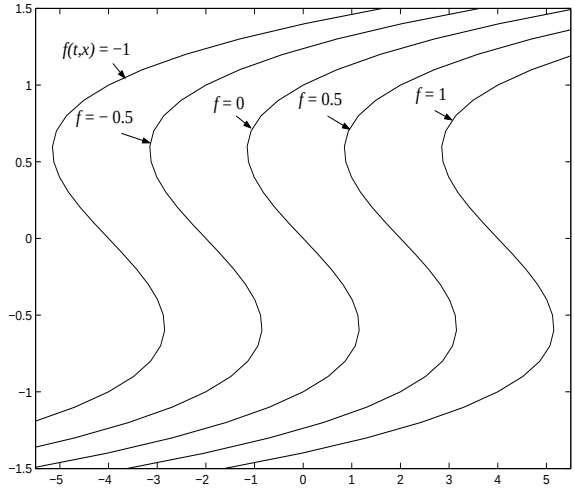
Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(15pts) (a) Verify whether or not $y(x) = -\cos(2x)$ is a solution to the equation: $y'' + xy = 2\sin(2x) - x\cos(2x)$.

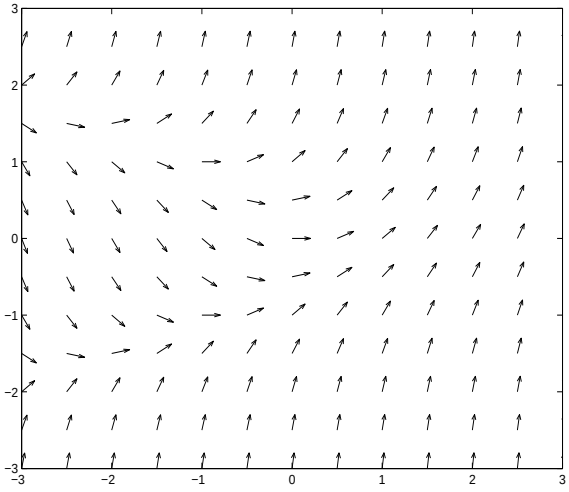
(b) Does $x^2 - \sin(x+y) = 1$ define an implicity solution to the equation $\dfrac{dy}{dx} = 2x\sec(x+y) - 1$?

2(10pts) (a) The isoclines: $f(t,x) = -1, -0.5, 0, 0.5, 1$ of a differential equation $x' = f(t,x)$ is given in figure (a). Sketch the slope field and the solution starting approximately at $(-5, 1)$.

(b) The vector field of an equation is given in figure (b). Sketch solutions that go through these points: (i) $(-3, 1)$, (ii) $(0, 0)$.



(a)



(b)

3(15pts) Use Euler's method to approximate the solution to the IVP: $y' = x^2 - y^2,\ y(0) = 1$ in the interval $[0, 1]$ by discretizing the interval into 4 equal parts. Sketch your approximating solution.

4(15pts) Find the solution to the initial value problem $x\dfrac{dx}{dt} = t + tx^2,\ x(0) = 1$.

5(15pts) Find a general solution to the linear equation $\dfrac{dx}{dt} + \dfrac{x}{t} = t^2$.

6(10pts) A brine solution containing 0.2 lb of salt per gallon is used to salinize a play pool containing 500 gallon of pure water. If the brine solution flows into the pool at 5 gallons per minute, and the well mixed solution flows out at the same rate, set up a differential equation for the amount of salt in the pool at any time. How much salt is in the pool 10 minutes after the solution starts to flow?

7(10pts) During the summer the temperature inside a van is 28°C after the driver turns off the air conditioner. If the temperature outside is 37°C and it takes 1 minute for the van to heat up to 30°C, write a differential equation for the temperature and estimate how many minutes will it take for the van to reach 35°C inside.

8(10pts) A parachutist weighing 75 kg steps out an airplane 3000 m about the ground. Assume the air resistance is proportional to the velocity with which she is falling and the proportionality is $b = 20$N-sec/m, write a differential equation for the falling parachutist and find her falling velocity 1 minutes later.

END