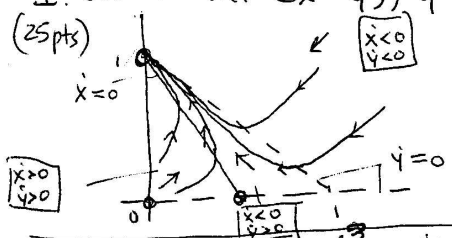
Math 221 Test 3 Key April 19 01

1. (a) $\dot{x} = x(1-2x-y),\ \dot{y} = y(1-y-x)$. competing spieces
(25pts)



(c) If $y(0) \neq 0$, y always dominates and x dies out.

2. (10pts) $\mathcal{L}\{f(t)\}(s) = \int_0^3 e^{-5t}e^{-2t}dt + \int_3^\infty e^{-st}\cdot 0\,dt = \int_0^3 e^{-7t}dt = -\frac{1}{7}e^{-7t}\Big|_0^3 = \frac{1}{7}(1-e^{-21})$

3. (15pts)
(a) $\mathcal{L}\{(t+1)^2\} = \mathcal{L}\{t^2+2t+1\} = \frac{2!}{s^3} + 2\cdot\frac{1!}{s^2} + \frac{1}{s} = \boxed{\frac{2}{s^3}+\frac{2}{s^2}+\frac{1}{s}}$

(b) $\mathcal{L}\{e^{-t}\cos(3t) - 2te^{5t}\} = \boxed{\frac{s+1}{(s+1)^2+9} - 2\frac{1}{(s-5)^2}}$

(c) $\mathcal{L}\{u(t-2)g(t)\} = e^{-2s}\mathcal{L}\{g(t+2)\} = \boxed{e^{-2s}\frac{1}{\sqrt{s^2+4}}}$

4. (20pts)
(a) $\frac{s}{2(s-3)^2+2} = \frac{1}{2}\frac{s-3+3}{(s-3)^2+1} = \frac{1}{2}\left(\frac{s-3}{(s-3)^2+1} + 3\frac{1}{(s-3)^2+1}\right) \xrightarrow{\mathcal{L}^{-1}} \boxed{\frac{1}{2}e^{3t}\cos t + \frac{3}{2}e^{3t}\sin t}$

(b) $\frac{1}{s^3} - \frac{1}{s^2+2} = \frac{1}{2}\frac{2}{s^3} - \frac{1}{\sqrt{2}}\frac{\sqrt{2}}{s^2+(\sqrt{2})^2} \xrightarrow{\mathcal{L}^{-1}} \boxed{\frac{1}{2}t^2 - \frac{1}{\sqrt{2}}\sin(\sqrt{2}t)}$

(c) $\frac{5}{s^2+4s+8} = \frac{5}{(s+2)^2+4} = \frac{5}{2}\frac{2}{(s+2)^2+2^2} \xrightarrow{\mathcal{L}^{-1}} \boxed{\frac{5}{2}e^{-2t}\sin(2t)}$

5. (15pts)
(a) $\frac{2s+1}{s^4+2s^3+2s^2} = \frac{2s+1}{s^2(s^2+2s+2)} = \boxed{\frac{A}{s}+\frac{B}{s^2}+\frac{Cs+D}{s^2+2s+2}}$

(b) $f(t) = 2 - 2u(t-1) + u(t-3) \xrightarrow{\mathcal{L}} \boxed{\frac{2}{s} - 2e^{-s}\frac{1}{s} + e^{-3s}\frac{1}{s}}$

6 (15pts) $\mathcal{L}\{y'\} - \mathcal{L}\{y\} = \mathcal{L}\{u(t-2)\} = e^{-2s}\frac{1}{s}$

$s\mathcal{L}\{y\} - y(0) - \mathcal{L}\{y\} = (s-1)\mathcal{L}\{y\} - 0 = \frac{e^{-2s}}{s}$. $\mathcal{L}\{y\} = \frac{1}{s(s-1)}$. $y(t) = \mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s(s-1)}\right\}$

$\frac{1}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1} = \frac{1}{s-1} - \frac{1}{s} \xrightarrow{\mathcal{L}^{-1}} e^t - 1$

$y(t) = \mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s(s-1)}\right\} = u(t-2)\left(\mathcal{L}^{-1}\left\{\frac{1}{s(s-1)}\right\}(t)\Big|_{t\to t-2}\right) = \boxed{u(t-2)(e^{t-2}-1)}$