Mathzz1 Test 2 Solu. Key Spring '01

1(15pts). $y''+4y'+5y=0$, $r^2+4r+5=0$, $r_{1,2}=\frac{-4\pm\sqrt{4^2-4\cdot5}}{2}=\frac{-4\pm i2}{2}=-2\pm i$

$y(t)=c_1e^{-2t}\cos t+c_2e^{-2t}\sin t$, $-2=y(0)=c_1$, $1=y'(0)=-2c_1+c_2$

$\to c_1=-2$, $c_2=1+2c_1=-3$ $\boxed{y(t)=-2e^{-2t}\cos t-3e^{-2t}\sin t}$

2(20pts). $y''+2y'+2y=0$, $r^2+2r+2=0$, $r_{1,2}=\frac{-2\pm\sqrt{4-4\cdot2}}{2}=-1\pm i$, $y_1(t)=e^{-t}\cos t$, $y_2=e^{-t}\sin t$

(a) $g(t)=1+t$, $y_p(t)=t^s[A+Bt]$, $s=0$ ok. $\boxed{y_p(t)=A+Bt}$

(b) $g(t)=e^{-t}\sin t$, $y_p(t)=t^s[Ae^{-t}\sin t+Be^{-t}\cos t]$, $s=1$, $\boxed{y_p(t)=Ate^{-t}\sin t+Bte^{-t}\cos t}$

(c) $g(t)=g_1(t)+g_2(t)$, $g_1(t)=1$, $y_{1p}=t^sA$, $s=0$. $g_2(t)=te^{3t}$, $y_{2p}=[(B+Ct)e^{3t}]t^s$, $s=1$

$\Rightarrow y_p(t)=y_{1p}(t)+y_{2p}(t)=\boxed{A+(B+Ct)e^{3t}}$

3(25pts). $y''-y=t^2$, $y''-y=0$, $r^2-1=(r+1)(r-1)=0$, $r_1=-1$, $r_2=1$, $y_1=e^{-t}$, $y_2=e^t$.

$g(t)=t^2$, $y_p(t)=t^s[A+Bt+Ct^2]$, $s=0$. $y_p'(t)=B+2Ct$, $y_p''=2C$.

$y_p''-y_p=2C-(A+Bt+Ct^2)=t^2$. $\Rightarrow -C=1$, $-B=0$, $2C-A=0$. $A=2C=-2$

$y_p=-2-t^2$, $\boxed{y(t)=c_1e^{-t}+c_2e^t-2-t^2}$

4(15pts) $\boxed{y(t)=c_1+c_2t+c_3e^{3t}+c_4e^{-3t}\cos t+c_5e^{-3t}\sin t}$

5(25pts). $y''-2y'+y=\frac{e^t}{t}$, $y_1(t)=e^t$, $y_2(t)=te^t$.

$W[y_1,y_2]=\begin{vmatrix}e^t & te^t\\ e^t & e^t+te^t\end{vmatrix}=e^{2t}$. $v_1(t)=-\int\frac{y_2g}{aW[y_1,y_2]}dt=-\int\frac{te^t\frac{e^t}{t}}{1\cdot e^{2t}}dt$

$=-\int dt=-t$. $v_2(t)=\int\frac{y_1g}{1\cdot W[y_1,y_2]}dt=\int\frac{e^t\cdot\frac{e^t}{t}}{e^{2t}}dt=\int\frac{1}{t}dt=\ln t$

$\Rightarrow y_p(t)=v_1(t)y_1(t)+v_2(t)y_2(t)=-te^t+te^t\ln t$, $\boxed{y_p(t)=te^t\ln t}$