

Math221 Test 1 Key Spring '01

1(15pts) $\ln y + xy = 5$, $\frac{1}{y} \frac{dy}{dx} + y + x \frac{dy}{dx} = 0$ $\frac{1+xy}{y} \frac{dy}{dx} = -y$, $\frac{dy}{dx} = \frac{-y^2}{1+xy}$

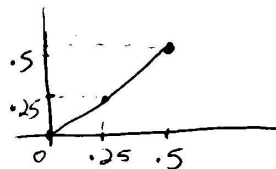
2(18pts) $\frac{dy}{dx} = \frac{x}{1+y}$, $y(0)=1$. $(1+y)dy = xdx$, $\frac{1}{2}(1+y)^2 = \frac{1}{2}x^2 + C$.

$(1+y)^2 = x^2 + C$, $(1+1)^2 = (1+y(0))^2 = 0^2 + C = C$. $C=4$, $(1+y)^2 = x^2 + 4$

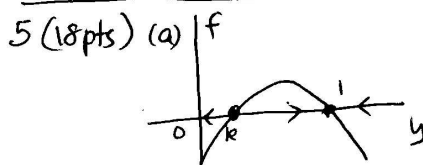
$1+y = \sqrt{x^2 + 4}$, $y = \sqrt{x^2 + 4} - 1$

3(18pts) $y' = f(x,y) = (x+1)(y+1)$. $x_0=0, y_0=0, y_n = y_{n-1} + h f(x_{n-1}, y_{n-1})$, $h=0.25$

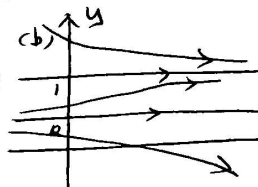
n	x_n	y_n	$y'_n = (x_n+1)(y_n+1)$
0	0	0	1
1	0.25	0.25	1.2548828
2	0.5	0.5637	



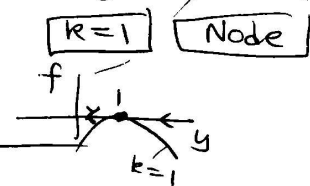
4(16pts) $y' - \frac{1}{1+t} y = 1$, $\mu(t) = e^{\int -\frac{1}{1+t} dt} = e^{-\ln(1+t)} = e^{\ln(1+t)^{-1}} = \frac{1}{1+t}$, $\frac{1}{1+t} y' - \frac{1}{(1+t)^2} y = \frac{1}{1+t}$
 $(\frac{1}{1+t} y)' = \frac{1}{1+t}$, $\frac{1}{1+t} y = \int \frac{1}{1+t} dt = \ln(1+t) + C$. $y(t) = (1+t) \ln(1+t) + C(1+t)$



Eg. pt. 1 - sink
k - source



(c) Eg. pts. 1, k,



6(15pts). $X(t)$ — amount of drug in gram at time t

$\frac{dX(t)}{dt} = 3 \text{ cm}^3/\text{sec} * 0.02 \text{ g/cm}^3 - \frac{X(t)}{125 \text{ cm}^3} * 3 \text{ cm}^3/\text{sec}$, $X(0)=0$

$\begin{cases} \frac{dX(t)}{dt} = 0.6 - \frac{3}{125} X(t) \\ X(0)=0 \end{cases}$