Math 221 Sec.3 **Test I** **Sept. 22, 2000**

Name:______________________________ Score:__________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(15 pts) Verify that $y = \cos x + x^2$ is a solution to the equation $y'' + y = x^2 + 2$.

2(15 pts) Find the solution to the initial value problem $x^2 \frac{dy}{dx} = 1 + y,\ y(1) = 1$.

3(14 pts) Determine the region in the ty plane in which the initial value problem

$$y' = \frac{2}{t - y},\ y(t_0) = y_0$$

has a unique solution and explain why. (*hint:* You need to verify the conditions of the Theorem for Existence and Uniqueness of Solution.)

4(17 pts) (a) Use the Euler method with step size $h = 0.2$ to find approximate values of the solution to the initial value problem $y' = \frac{1}{x}(y^3 + 1), y(1) = -1$ at $x = 1.2, 1.4, 1.6$.
(b) Plot your approximating solution in the xy plane.

5(15 pts) Find the general solution to the equation $ty' + 2y = t^2$.

6(12 pts) (a) Sketch the phase line

$$\frac{dy}{dt} = f(y) = y^3 + y.$$

(b) Classify each equilibrium point as sink, source, or node,
(b) Sketch a qualitative portrait for the solutions, including the ones through $y(0) = -2, -0.5, 0.5, 2$.

7(12 pts) A brine solution of 0.2 kg/L concentration flows at a constant rate of 3 L/min into a large tank that initially held 50 L of pure water. The solution inside the tank is kept well stirred and flows out of the tank at a slower rate of 2 L/min. Write done a differential equation with initial condition that describe the amount of salt $x(t)$ at time t. If the tank has a capacity of 100 L, then for what range of time t is your equation valid?

END