Matlab Lab 4

Example 1 (System of Linear Equations) To find the general solution to the system of equations

$$x' = x + 2y$$
$$y' = -2x + y$$

type at the command line

```
>> 'Your Name Here '

>> syms x y t
>> [x,y]=dsolve('Dx=x+2*y','Dy=-2*x+1*y')
```

and Matlab returns the following:

```
x =

C2*cos(2*t)*exp(t) + C1*sin(2*t)*exp(t)


y =

C1*cos(2*t)*exp(t) - C2*sin(2*t)*exp(t)
```

To solve the same equation with the initial condition $x(0) = 1, y(0) = 2$, type the following two lines and the output follows:

```
>> syms x y t
>> [x,y]=dsolve('Dx=x+2*y','Dy=-2*x+1*y','x(0)=1','y(0)=2')

x =

cos(2*t)*exp(t) + 2*sin(2*t)*exp(t)


y =

2*cos(2*t)*exp(t) - sin(2*t)*exp(t)
```

Example 2 (Systems of Higher Orders) To solve the system of higher order of equations

$$x'' = 7x - 6y$$
$$y' = x$$

type at the command prompt the following:

```
>> rand(1,4)

ans =

    0.9575    0.9649    0.1576    0.9706

>> syms x y t
>> [x,y]=dsolve('D2x=7*x-6*y','Dy=x')

x =

(exp(-3*t)*(3*C1*exp(5*t) - 2*C2 + 6*C3*exp(4*t)))/6


y =

(exp(-3*t)*(4*C2 + 9*C1*exp(5*t) + 36*C3*exp(4*t)))/36
```

Example 3 (Laplace Transform) To find the Laplace transform of function

$$f(t) = 2t + 3\sin(2t) + e^{t}u(t-2)$$

type at the command prompt

```
>> tic
>> syms t s
>> f=2*t+3*sin(2*t)+exp(t)*heaviside(t-2)

f =

2*t + 3*sin(2*t) + heaviside(t - 2)*exp(t)

>> F=laplace(f)

F =

6/(s^2 + 4) + 2/s^2 + (exp(-2*s)*exp(2))/(s - 1)
```

Example 4 (Laplace Inverse Transform) To find the inverse Laplace transform of function

$$F(s) = \frac{e^{(-2s)}(s-5)}{s(s+2)^2}$$

type at the command prompt

```
>> syms t s
>> F=exp(-2*s)*(s-5)/(s*(s+2)^2);
>> f=ilaplace(F);
>> toc
Elapsed time is 483.629303 seconds.
>> simplify(f)

ans =

-(heaviside(t - 2)*exp(4 - 2*t)*(5*exp(2*t - 4) - 14*t + 23))/4
```

Example 5 (Laplace Transform for ODE) To solve this differential equation

$$y'' - 6y' + 8y = 0, \;\; y(0) = -1, y'(0) = 2$$

by the Laplace transform method, type at the command prompt

```
>> syms s t Y
>> ode='D(D(y))(t)-6*D(y)(t)+8*y(t)=0'

ode =

D(D(y))(t)-6*D(y)(t)+8*y(t)=0

>> Lode=laplace(ode,t,s);
>> eqn=subs(Lode,{'laplace(y(t),t,s)','y(0)','D(y)(0)'},{Y,-1,2})

eqn =

8*Y + s - 6*Y*s + Y*s^2 - 8 == 0

>> Y=solve(eqn,Y)

Y =

-(s - 8)/(s^2 - 6*s + 8)

>> y=ilaplace(Y,s,t)

y =

2*exp(4*t) - 3*exp(2*t)
```

Example 6 (Resonance Phenomenon) To solve this differential equation

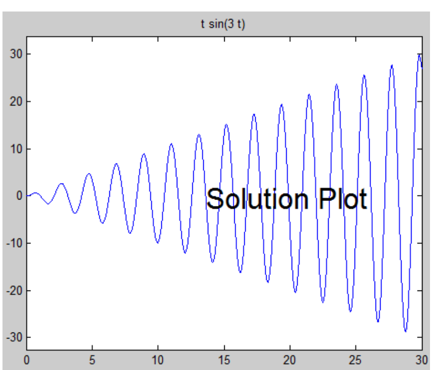
$$y'' + 9y = 6\cos(3t), \;\; y(0) = 0, y'(0) = 0$$

by the Laplace transform method, type at the command prompt

```
>> syms s t Y
>> rand(1,4)

ans =

    0.4218    0.9157    0.7922    0.9595

>> ode='D(D(y))(t)+9*y(t)=6*cos(3*t)'

ode =

D(D(y))(t)+9*y(t)=6*cos(3*t)

>> Lode=laplace(ode,t,s);
>> eqn=subs(Lode,{'laplace(y(t),t,s)','y(0)','D(y)(0)'},{Y,0,0})

eqn =

Y*s^2 + 9*Y == (6*s)/(s^2 + 9)

>> Y=solve(eqn,Y)

Y =

(6*s)/(s^2 + 9)^2

>> y=ilaplace(Y,s,t)

y =

t*sin(3*t)
```

We can plot the solution to see that the amplitude increases with time:

```
>> ezplot(y,[0,30])
```



Example 7 (Piecewise Forcing) Use Matlab to solve this differential equation with piecewise forcing

$$y'' + 4y = 1 - u(t - \pi), \;\; y(0) = 0, y'(0) = 0$$

by the Laplace transform method. Show all commands together with the simplified result:

```
>> simplify( y)

ans =

((heaviside(t - pi) - 1)*(cos(2*t) - 1))/4
```

Example 8 (Impulse Forcing) To solve this differential equation with impulse forcing

$$y'' + 2y' + y = 3\delta(t - 2), \;\; y(0) = 2, y'(0) = 1$$

by the Laplace transform method, type at the command prompt

```
>> syms s t Y
>> ode='D(D(y))(t)+ 2*D(y)(t)+y(t)  =3*dirac(t -2)'

ode =

D(D(y))(t)+ 2*D(y)(t)+y(t)  =3*dirac(t -2)

>> Lode=laplace(ode,t,s);
>> eqn=subs(Lode,{' laplace(y(t),t,s)','y(0)','D(y)(0)'},{Y,2,1})

eqn =

Y - 2*s + 2*Y*s + Y*s^2 - 5 == 3*exp(-2*s)

>> tic;
>> Y=solve(eqn,Y)

Y =

(2*s + 3*exp(-2*s) + 5)/(s^2 + 2*s + 1)

>> toc
Elapsed time is 17.398847 seconds.
>> y=ilaplace(Y, s,t)

y =

2*exp(-t) + 3*t*exp(-t) + 3*heaviside(t - 2)*exp(2 - t)*(t - 2)
```

(See Lab 1 for instruction to prepare your hand-in work.)

End Lab 4