Matlab Lab 4

**Example 1 (System of Linear Equations)** To find the general solution to the system of equations

\[
\begin{align*}
    x' &= x + 2y \\
    y' &= -2x + y
\end{align*}
\]

type at the command line

```matlab
>> 'Your Name Here '
>> syms x y t 
>> [x,y]=dsolve('Dx=x+2*y','Dy=-2*x+1*y')
```

and Matlab returns the following:

\[
\begin{align*}
    x &= C2\cos(2t)\exp(t) + C1\sin(2t)\exp(t) \\
    y &= C1\cos(2t)\exp(t) - C2\sin(2t)\exp(t)
\end{align*}
\]

To solve the same equation with the initial condition \(x(0) = 1, y(0) = 2\), type the following two lines and the output follows:

```matlab
>> syms x y t 
>> [x,y]=dsolve('Dx=x+2*y','Dy=-2*x+1*y','x(0)=1','y(0)=2')
```

\[
\begin{align*}
    x &= \cos(2t)\exp(t) + 2\sin(2t)\exp(t) \\
    y &= 2\cos(2t)\exp(t) - \sin(2t)\exp(t)
\end{align*}
\]

**Example 2 (Systems of Higher Orders)** To solve the system of higher order of equations

\[
\begin{align*}
    x'' &= 7x - 6y \\
    y' &= x
\end{align*}
\]

type at the command prompt the following:
Example 3 (Laplace Transform) To find the Laplace transform of function
\[ f(t) = 2t + 3\sin(2t) + e^t u(t - 2) \]
type at the command prompt

```matlab
>> tic
>> syms t s
>> f=2*t+3*sin(2*t)+exp(t)*heaviside(t-2)
```

\[ f = 
2t + 3\sin(2t) + \text{heaviside}(t - 2)\exp(t) 
\]

```matlab
>> F=laplace(f)
```

\[ F = 
\frac{6}{s^2 + 4} + \frac{2}{s^2} + \frac{(\exp(-2s)\exp(2))}{(s - 1)} 
\]

Example 4 (Laplace Inverse Transform) To find the inverse Laplace transform of function
\[ F(s) = \frac{e^{-2s}(s - 5)}{s(s + 2)^2} \]
type at the command prompt
Example 5 (Laplace Transform for ODE) To solve this differential equation
\[
y'' - 6y' + 8y = 0, \quad y(0) = -1, y'(0) = 2
\]
by the Laplace transform method, type at the command prompt

```matlab
>> syms s t Y
>> ode='D(D(y))(t)-6*D(y)(t)+8*y(t)=0'

ode =

D(D(y))(t)-6*D(y)(t)+8*y(t)=0

>> Lode=laplace(ode,t,s);
>> eqn=subs(Lode,'laplace(y(t),t,s)',"y(0)",'D(y)(0)',{Y,-1,2})

eqn =

8*Y + s - 6*Y*s + Y*s^2 - 8 == 0

>> Y=solve(eqn,Y)

Y =

-(s - 8)/(s^2 - 6*s + 8)

>> y=ilaplace(Y,s,t)

y =

2*exp(4*t) - 3*exp(2*t)
```
Example 6 (Resonance Phenomenon) To solve this differential equation
\[ y'' + 9y = 6\cos(3t), \ y(0) = 0, y'(0) = 0 \]
by the Laplace transform method, type at the command prompt

```matlab
>> syms s t Y
>> rand(1,4)

ans = 
    0.4218  0.9157  0.7922  0.9595

>> ode='D(D(y))(t)+9*y(t)=6*cos(3*t)'

ode =

D(D(y))(t)+9*y(t)=6*cos(3*t)

>> Lode=laplace(ode,t,s);
>> eqn=subs(Lode,'laplace(y(t),t,s)','y(0)','D(y)(0)',{Y,0,0})

eqn =

Y*s^2 + 9*Y == (6*s)/(s^2 + 9)

>> Y=solve(eqn,Y)

Y =

(6*s)/(s^2 + 9)^2

>> y=ilaplace(Y,s,t)

y =

t*sin(3*t)
```

We can plot the solution to see that the amplitude increases with time:

```matlab
>> ezplot(y,[0,30])
```

![Solution Plot](image.png)
Example 7 (Piecewise Forcing) Use Matlab to solve this differential equation with piecewise forcing

\[ y'' + 4y = 1 - u(t - \pi), \quad y(0) = 0, y'(0) = 0 \]

by the Laplace transform method. Show all commands together with the simplified result:

```matlab
>> simplify(y)
ans =
((heaviside(t - pi) - 1)*(cos(2*t) - 1))/4
```

Example 8 (Impulse Forcing) To solve this differential equation with impulse forcing

\[ y'' + 2y' + y = 3\delta(t - 2), \quad y(0) = 2, y'(0) = 1 \]

by the Laplace transform method, type at the command prompt

```matlab
>> sym s t Y
>> ode='D(D(y))(t) + 2*D(y)(t)+y(t) =3*dirac(t-2)'

ode =
D(D(y))(t) + 2*D(y)(t)+y(t) =3*dirac(t-2)

>> Lode=laplace(ode,t,s);
>> eqn=subs(Lode,{' laplace(y(t),t,s)' ,'y(0)' ,'D(y)(0)'},{Y,2,1})
eqn =
Y - 2*s + 2*Y*s + Y*s^2 - 5 == 3*exp(-2*s)

>> tic;
>> Y=solve(eqn,Y)

Y =
(2*s + 3*exp(-2*s) + 5)/(s^2 + 2*s + 1)

>> toc
Elapsed time is 17.398847 seconds.
>> y=ilaplace(Y, s,t)
y =
2*exp(-t) + 3*t*exp(-t) + 3*heaviside(t - 2)*exp(2 - t)*(t - 2)
```

(See Lab 1 for instruction to prepare your hand-in work.)

End Lab 4