Math 208 Quiz 5

Name:_____ Score:_____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1. (*4*) Set up an iterated integral in the spherical coordinate for the triple integral $\iiint_Q xz\,dV$ where Q is the solid bound by $z = \sqrt{x^2 + y^2}$ and $z = 2$. (Do not evaluate the integral.)

2. (*6*) (a) Use the Component Test to verify that the vector field $\vec{F}(x, y, z) = \langle yz + 2, xz, xy \rangle$ is a conservative field.

   (b) Find a potential function ϕ for the vector field.

3. (*2*) Let $\vec{F}(x,y,z) = \nabla\phi(x,y,z)$ be a gradient vector field with a potential function $\phi(x,y,z) = xyz^2 + x$. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ where C is the helix $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + 2t\vec{k}$ for $0 \leq t \leq 2\pi$.

4. (*4*) Find the work done by the force $\vec{F}(x,y) = \langle x, 3\rangle$ along the upper half circle $C : x^2 + y^2 = 4$ from $(2,0)$ to $(-2,0)$.

5. (*4*) Use Green's Theorem to evaluate the line integral $\oint_C (y^2 - 2x)dx + x^2 dy$ where C is the boundary of a triangle with vertexes $(1,0), (2,0), (2,3)$, going counterclockwise.

END