Name:

Score: \_\_\_\_\_

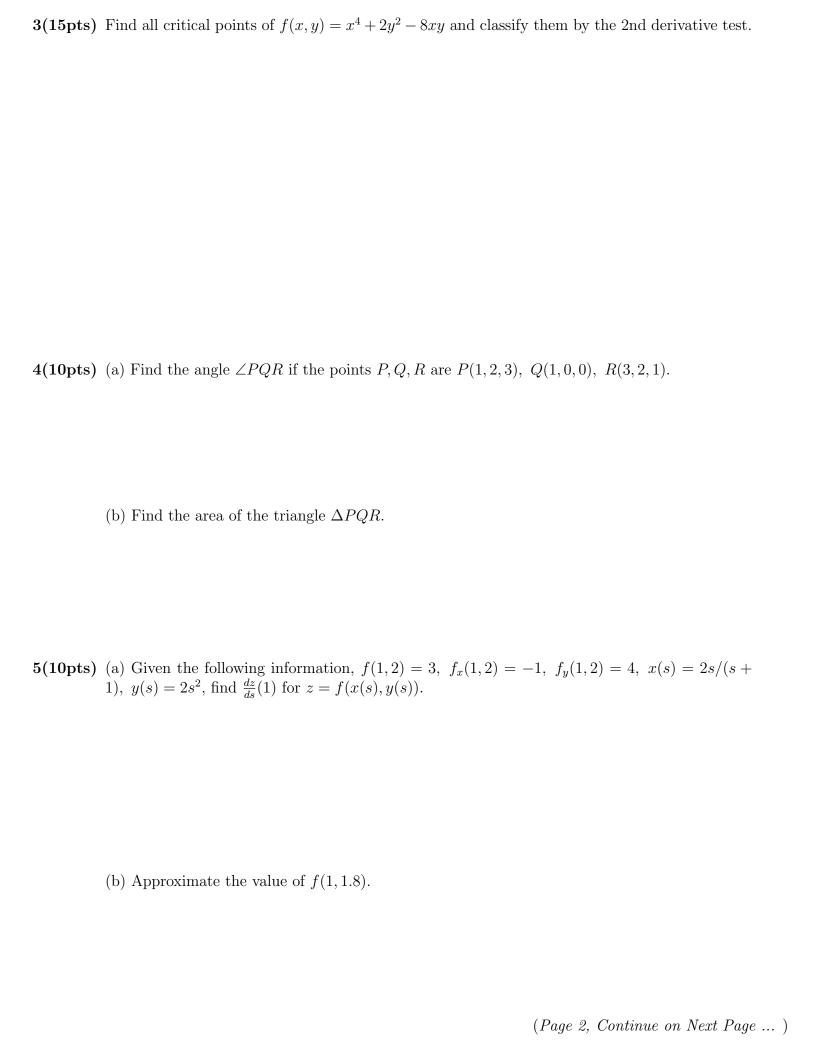
Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

**1(20pts)** (a) Find  $\left(\frac{df}{ds}\right)_{\mathbf{n}, P_0}$  where f(x, y) = 2xy,  $P_0 = (2, -1)$  and  $\mathbf{n}$  is the direction of vector  $\langle -12, 5 \rangle$ .

- (b) Find the direction at which the function z = f(x, y) increases most rapidly at the point  $P_0$ .
- (c) Find a point (x, y) at which the minimal rate of change of the function z = f(x, y) is -2.

**2(10pts)** (a) It is given that z can be solved as a function of x, y from the equation  $yz^3 - 2xz - e^{xy} = 1$  at the point (1, 0, -1). Use implicit differentiation to find  $\frac{\partial z}{\partial x}(1, 0)$ .

(b) Find an equation for the tangent plane at the point (1,0,-1) to the surface  $yz^3-2xz-e^{xy}=1$ .



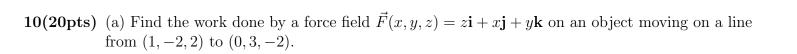
6(15pts) Find the limit if exists, or use the 2-path rule to show it does not.

(a) 
$$\lim_{(x,y)\to(1,0)} \frac{x+2y-x^2-2xy}{x^2-xy-x+y}$$

(b) 
$$\lim_{(x,y)\to(0,2)} \frac{y+x-2}{xy}$$

**7(15pts)** Use the Lagrange multiplier method to find the shortest distance from the point (1, 4, 1) to the cylindrical paraboloid,  $z = \frac{1}{2}y^2$ .

6(13pts)	hour.  (a) Find the velocity when the rocket reaches its maximal height.
	(b) Find the time when the rocket reaches its maximal height.
	(c) Find the maximal height.
$9(15 \mathrm{pts})$	A particle is moving along a curve $C: \vec{r}(t) = \langle t^2, t \sin(\pi t), \sqrt{3t+1} \rangle$ .  (a) Find a parameter equation for the tangent line to the path when $t=1$ .
	(b) Find an equation of the plane that is perpendicular to the path at the point when $t=1$ .
	(c) Find the acceleration $\vec{r}''(1)$ .

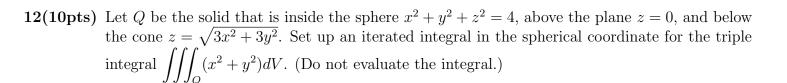


(b) Let C be the close path consisting of the interval [-2,2] on the x-axis and the upper semi-circle  $x^2 + y^2 = 4$ . Find the line integral  $\oint_C (2-3x)dx + y^2dy$ .

**11(20pts)** (a) Use the Component Test to show that the force field  $\vec{F}(x,y) = \langle 2xy + 2, x^2 - 1 \rangle$  is a conservative vector field.

(b) Find a potential function  $\phi(x,y)$  for  $\vec{F}$ .

(c) Find the work done by the force field  $\vec{F}$  on a particle moving from point (0,2) to (3,1) along the curve  $C: x+y^2=4$ .



**13(10pts)** Let Q be the solid in the first octant that is bounded by the surface  $x + y + z^2 = 9$ . Set up an iterated integral for  $\iiint_Q f(x,y,z)dV$  in the order of dxdydz.

**14(15pts)** Let  $\vec{F} = \langle x, y, z \rangle$  and S be the paraboloid  $z = 4 - x^2 - y^2$  above the xy-plane, with downward orientation. Set up an iterated integral in polar coordinate for the flux of  $\vec{F}$  through S, and then find the flux.