Math 208 Summer II 2007 **Final Exam**

Name: ______________________ **Score:** ______________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(20pts) (a) Find $\left(\dfrac{df}{ds}\right)_{\mathbf{n},\ P_0}$ where $f(x, y) = 2xy$, $P_0 = (2, -1)$ and $\mathbf{n}$ is the direction of vector $\langle -12, 5\rangle$.

(b) Find the direction at which the function $z = f(x, y)$ increases most rapidly at the point P_0.

(c) Find a point (x, y) at which the minimal rate of change of the function $z = f(x, y)$ is -2.

2(10pts) (a) It is given that z can be solved as a function of x, y from the equation $yz^3 - 2xz - e^{xy} = 1$ at the point $(1, 0, -1)$. Use implicit differentiation to find $\dfrac{\partial z}{\partial x}(1, 0)$.

(b) Find an equation for the tangent plane at the point $(1, 0, -1)$ to the surface $yz^3 - 2xz - e^{xy} = 1$.

3(15pts) Find all critical points of $f(x, y) = x^4 + 2y^2 - 8xy$ and classify them by the 2nd derivative test.

4(10pts) (a) Find the angle $\angle PQR$ if the points P, Q, R are $P(1, 2, 3)$, $Q(1, 0, 0)$, $R(3, 2, 1)$.

(b) Find the area of the triangle ΔPQR.

5(10pts) (a) Given the following information, $f(1, 2) = 3$, $f_x(1, 2) = -1$, $f_y(1, 2) = 4$, $x(s) = 2s/(s + 1)$, $y(s) = 2s^2$, find $\frac{dz}{ds}(1)$ for $z = f(x(s), y(s))$.

(b) Approximate the value of $f(1, 1.8)$.

6(15pts) Find the limit if exists, or use the 2-path rule to show it does not.

(a) $$\lim_{(x,y)\to(1,0)} \frac{x+2y-x^2-2xy}{x^2-xy-x+y}$$

(b) $$\lim_{(x,y)\to(0,2)} \frac{y+x-2}{xy}$$

7(15pts) Use the Lagrange multiplier method to find the shortest distance from the point $(1,4,1)$ to the cylindrical paraboloid, $z=\frac{1}{2}y^2$.

8(15pts) A rocket is launched at a 45° angle with the ground level with a initial velocity of 200 miles per hour.

(a) Find the velocity when the rocket reaches its maximal height.

(b) Find the time when the rocket reaches its maximal height.

(c) Find the maximal height.

9(15pts) A particle is moving along a curve $C : \vec{r}(t) = \langle t^2, t\sin(\pi t), \sqrt{3t+1}\rangle$.

(a) Find a parameter equation for the tangent line to the path when $t = 1$.

(b) Find an equation of the plane that is perpendicular to the path at the point when $t = 1$.

(c) Find the acceleration $\vec{r}''(1)$.

10(20pts) (a) Find the work done by a force field $\vec{F}(x, y, z) = z\mathbf{i} + x\mathbf{j} + y\mathbf{k}$ on an object moving on a line from $(1, -2, 2)$ to $(0, 3, -2)$.

(b) Let C be the close path consisting of the interval $[-2, 2]$ on the x-axis and the upper semi-circle $x^2 + y^2 = 4$. Find the line integral $\oint_C (2 - 3x)dx + y^2 dy$.

11(20pts) (a) Use the Component Test to show that the force field $\vec{F}(x, y) = \langle 2xy + 2, x^2 - 1 \rangle$ is a conservative vector field.

(b) Find a potential function $\phi(x, y)$ for $\vec{F}$.

(c) Find the work done by the force field $\vec{F}$ on a particle moving from point $(0, 2)$ to $(3, 1)$ along the curve $C : x + y^2 = 4$.

12(10pts) Let Q be the solid that is inside the sphere $x^2 + y^2 + z^2 = 4$, above the plane $z = 0$, and below the cone $z = \sqrt{3x^2 + 3y^2}$. Set up an iterated integral in the spherical coordinate for the triple integral $\iiint_Q (x^2 + y^2)dV$. (Do not evaluate the integral.)

13(10pts) Let Q be the solid in the first octant that is bounded by the surface $x + y + z^2 = 9$. Set up an iterated integral for $\iiint_Q f(x, y, z)dV$ in the order of $dxdydz$.

14(15pts) Let $\vec{F} = \langle x, y, z \rangle$ and S be the paraboloid $z = 4 - x^2 - y^2$ above the xy-plane, with downward orientation. Set up an iterated integral in polar coordinate for the flux of $\vec{F}$ through S, and then find the flux.

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2 Bonus Points: The gradient of a function is : (a) a number, (b) a vector, (c) a surface, (d) all above.