Exam 1 Topics Review

1. Vectors in $\mathbb{R}^2$
   (a) Vectors and scalars. The magnitude and direction of a vector.
   (b) Graphical vector operations: addition, subtraction, scalar multiplication.
   (c) Vectors in component form. Vectors in polar form.
   (d) Vector algebra with components: addition, subtraction, scalar multiplication.
   (e) The zero vector. The standard basis vectors $\vec{i}, \vec{j}$. Vector as a linear combination of the standard basis vectors.
2. Vectors in $\mathbb{R}^3$
   (a) Graphical vector operations: addition, subtraction, scalar multiplication.
   (b) The standard basis vectors $\vec{i}, \vec{j}, \vec{k}$. Vector as a linear combination of the standard basis vectors.
   (c) Magnitude, length of a vector.
   (d) Parallel vectors. Unit vectors.
   (e) Vector algebra with components: addition, subtraction, scalar multiplication.
3. The dot product of vectors.
   (a) Definition of the dot product in component form.
   (b) The cosine formula for the dot product.
   (c) Basic properties of the dot product in terms of algebraic operations.
   (d) Angle between two vectors. Orthogonal vectors.
   (e) Dot production condition for orthogonal vectors.
   (f) Component of a vector in another. Projection of a vector in another.
4. The cross product of vectors in $\mathbb{R}^3$.
   (a) $2 \times 2, 3 \times 3$ determinants.
   (b) Definition of the cross product in component form.
   (c) The right-hand rule for the direction of the cross product.
   (d) Magnitude of the cross product. The area of a parallelogram, triangle. The volume of a parallelpiped.
   (e) Basic properties of the cross product in terms of algebraic operations.
5. Lines $\mathbb{R}^3$.
   (a) Direction of a line. Parametric equations of lines. Symmetrical equations of lines.
   (b) Equation of a line through two points.
   (c) Find intersection of two lines.
   (d) Distance from a point to a line. Distance between two lines.
6. Planes $\mathbb{R}^3$.
   (a) Normal direction of of a plane.
   (b) Equation of a plane through a point with a normal vector.
   (c) Equation of a plane through 3 points.
   (d) Determine normal vectors for a plane using cross product.
   (e) Condition of two parallel planes.
   (f) Distance from a point to a plane. Distance between a line and a plane. Distance between 2 planes.
7. Vector-valued functions.
   (a) Functions from $\mathbb{R}$ to $\mathbb{R}^2$ or $\mathbb{R}^3$.

(b) Equations of curves. Parameterized equations of curves. Orientation of a curve by its parameterization.

(c) Parameterizations of simple curves: lines, circles, ellipses, parabolas, helixes.

(d) Calculus of vector-valued functions: differentiation and integration.

(e) Tangent vector of a curve.

(f) Arc length of a curve.

8. Motion in space.

   (a) Position vector, velocity vector, and acceleration vector.

   (b) Unite tangent vector. The principal unit normal vector.

   (c) Arc length parameterization. Speed. Curvature of a curve.

   (d) Tangential component of acceleration, normal component of acceleration.

   (e) Acceleration vector decomposed in its tangential and principal normal components.

   (f) Newton's second law of motion. Newton's law of gravity. Projectile motion.

9. Function of several variables.

   (a) Real valued functions taking $\mathbb{R}^2$ or $\mathbb{R}^3$ to $\mathbb{R}$. Domain and range of a function.

   (b) Graph of a 2 variable function. Level curves, contours, trances of a function of 2 variables.

   (c) Level surfaces of functions of 3 variables.

10. Limits of functions of several variables.

    (a) Evaluation of limits with singularities.

    (b) Non-existence of limit by 2-path rule.

    (c) Continuity of functions.