Math 208 Summer II 2006 **Exam 1**

Name: ____________________ **Score:** ____________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(10pts) Three points in the space are given: $P(0,1,2), Q(1,2,3), R(-1,-1,1)$.

(a) Find the angle between the vectors $\vec{PQ}, \vec{PR}$.

2.8

(b) Find the projection of vector $\vec{PR}$ on $\vec{PQ}$: $\mathbf{proj}_{\vec{PQ}}\vec{PR}$.

$\frac{-4}{3}\langle 1,1,1\rangle$

2(6pts) Find the equation of the plane that goes through the point $(1,0,-2)$ and is parallel to another plane: $x+2y+3z+4=0$.

$x+2y+3z+5=0$

3(6pts) Sketch the surface of the equation $x+y^2+2z^2-1=0$, showing a few appropriate traces.

Elliptical paraboloid, vertex at $(1,0,0)$, open to the negative x-axis, with x-trace being ellipses: $y^2+2z^2=1-x$ for $x\le 1$.

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4(18pts) Three points in the space are given: $P(0,1,2), Q(2,0,1), R(-1,-1,1)$.

(a) Find the equation of the plane containing the points.

$-x + 3y - 5z + 7 = 0$

(b) Find the area of the triangle with these points as its vertexes.

$\frac{\sqrt{35}}{2}$

(c) Find the parametric equations of the line through point P and perpendicular to the plane.

$x = -t,\ y = 1 + 3t,\ z = 2 - 5t.$

5(7pts) The velocity of a moving particle is given as $\vec{v}(t) = \langle t, 2t, e^{2t} \rangle$. Find its position $\vec{r}(t)$ if $\vec{r}(0) = \langle 0, 1, 1 \rangle$.

$\vec{r}(t) = \langle \frac{1}{2}t^2, t^2 + 1, \frac{1}{2}(e^{2t} + 1) \rangle.$

6(14pts) Find the limit if exists, or show it does not exist by the 2-path rule.

(a) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{\cos(xy)}{1 + x^2 + y^2} = 0$

(b) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{xy}{2x^2 + y^3}$

Does not exist. On path $x = 0$, the path limit is 0. On $y = x$, the path limit is $1/2$. So the 2-path rule applies.

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7(15pts) Consider the curve given by $\vec{r}(t) = \langle t, \cosh t \rangle$.

$\sinh t = \frac{e^t - e^{-t}}{2}$, $\cosh t = \frac{e^t + e^{-t}}{2}$
$\tanh t = \frac{\sinh t}{\cosh t}$, $\coth t = \frac{\cosh t}{\sinh t}$
$\text{sech} t = \frac{1}{\cosh t}$, $\text{csch} t = \frac{1}{\sinh t}$
$\cosh^2 t - \sinh^2 t = 1$, $\tanh^2 t + \text{sech}^2 t = 1$
$\sinh' t = \cosh t$, $\cosh' t = \sinh t$
$\tanh' t = \text{sech}^2 t$, $\text{sech}' t = -\text{sech} t \tanh t$
$\coth' t = -\text{csch}^2 t$, $\text{csch}' t = -\text{csch} t \coth t$

(a) Find the unit tangent vector $\vec{T}(t)$.

Use the formulas. $\vec{T}(t) = (\text{sech} t, \tanh t)$

(b) Find the unit principal normal vector $\vec{N}(t)$.

$\vec{N}(t) = (-\tanh t, \text{sech} t)$

(c) Find the curvature.

Use either $\kappa = \frac{\|\vec{T}'(t)\|}{\|\vec{r}'(t)\|}$ or $\kappa = \frac{\|\vec{r}'(t) \times \vec{r}''(t)\|}{\|\vec{r}'(t)\|^3}$ to get $\kappa = \text{sech}^2 t$

8(6pts) Find the distance from the point $(1, 1, 2)$ to the line which goes through $(1, 0, 1)$ and $(3, 2, -1)$.

Let $P = (1, 0, 1), Q = (3, 2, -1),\ R = (1, 1, 2)$. Then $d = \frac{\|\vec{PQ} \times \vec{PR}\|}{\|\vec{PQ}\|} = \sqrt{2}$

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9(18pts) At an instance the following are given for a particle in motion: The acceleration $\vec{a} = (0, 3, 4)$, the velocity $\vec{v} = (-1, 0, 1)$. Find the following: (*Hint*: use the relation $\vec{a} = a_T\vec{T} + a_N\vec{N}$.)

(a) The speed $\frac{ds}{dt}$ at the instance.

$\|\vec{v}\| = \sqrt{2}$

(b) The tangential component of the acceleration a_T at the instance.

$\vec{T} = \vec{v}/\|\vec{v}\| = \frac{1}{\sqrt{2}}(-1, 0, 1)$. $a_T = \vec{a} \cdot \vec{T} = \frac{4}{\sqrt{2}}$

(c) The normal component of the acceleration a_N at the instance.

$\|\vec{a}\| = 5$. $a_N = \sqrt{\|\vec{a}\|^2 - a_T^2} = \sqrt{17}$

(d) The trajectory's curvature κ at the instance.

Use either $\kappa = \frac{\|\vec{r}' \times \vec{r}''\|}{\|\vec{r}'\|^3} = \frac{\|\vec{v} \times \vec{a}\|}{\|\vec{v}\|^3} = \frac{\sqrt{17}}{2}$, or use $\kappa = a_N/(ds/dt)^2$.

(d) The principal normal unit vector $\vec{N}$ at the instance.

$\vec{N} = \frac{1}{a_N}(\vec{a} - a_T\vec{T}) = \frac{1}{\sqrt{17}}(2, 3, 2)$

(e) The binormal unit vector $\vec{B}$ at the instance.

$\vec{B} = \vec{T} \times \vec{N} = \frac{1}{\sqrt{34}}(-3, 4, -3)$

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2 Bonus Points: Calculus was invented in: **(a) The 17th century**. (b) The 18th century. (c) The 19th century. (d) The 20th century. (... *The End*)