Math 208 Quiz 4

Name:______________________________ Score:____________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1. (*4*) Find the critical points and use the second derivative test to determine their type as local maximum, local minimum, saddle for the function $z = f(x, y) = x^3 - 3xy + y^3$.

   Solve $f_x = 3x^2 - 3y = 0, f_y = -3x + 3y^2 = 0$ for critical points to get $y = x^2$ and $-x + x^4 = x(-1 + x^3) = 0$, which give rise to two c.pts: $(0, 0), (1, 1)$. To use the 2nd derivative test, $f_{xx} = 6x, f_{xy} = -3, f_{yy} = 6y$, and the discriminant $D = f_{xx}f_{yy} - f_{xy}^2 = 36xy - 9$.

c.pt	$(0,0)$	$(1,1)$
D	$-$	$+$
f_{xx}	/	$+$
Classification	saddle	local min.

2. (*4*) Use Lagrange multiplier method to find the extrema of the function $z = f(x, y) = x + 2y$ subject to the constraint $x^2 + y^2 = 5$.

   Let $g(x, y) = x^2 + y^2$ and $g(x, y) = 5$ be the constraint. Then solve

$$\begin{cases} \nabla f(x,y) = \lambda \nabla g(x,y) \\ g(x,y) = 5 \end{cases}$$

   for constrained critical points. It results in these equations in components: $1 = \lambda 2x, 2 = \lambda 2y, x^2 + y^2 = 5$. Eliminate λ by taking the quotient of the first 2 equations to get $1/2 = x/y$ or $y = 2x$. Couple it with the constraint to reduce the equations to $5 = x^2 + (2x)^2 = 5x^2$ and $x = \pm 1$. Back substitute to get the constrained c.pts: $(1, 2), (-1, -2)$. Compare their f values to conclude below

c.c.pt	$(1,2)$	$(-1,-2)$
f	5	-5
Classification	constrained max.	constrained min.

3. (*4*) Evaluate the iterated integral $\int_0^1 \int_{2x}^2 (1 + 2y)dydx$.

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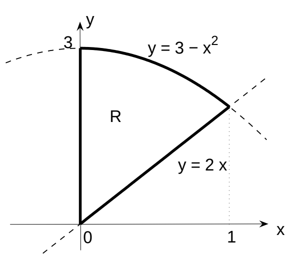
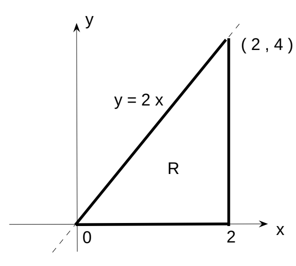
4. (*4*) Sketch the region R for the double integral $\int_0^1 \int_{2x}^{3-x^2} f(x, y)dydx$.



Problem 4



Problem 5

5. (*4*) Sketch the region R for the double integral $\int_0^2 \int_0^{2x} f(x, y)dydx$ and reverse the order of the iterated integral.

   $\boxed{\int_0^4 \int_{y/2}^2 f(x,y)dxdy}$

END