Math 208 Spring 2020 **Exam 1.a** **Print Your 8 Digits NU ID:** _____

Print Your Name Legibly: _____ **Score:** _____

Instructions: You must show supporting work to receive full and partial credits. Textbook, notes, cheat sheets, calculators are not allowed.

1(12pts) For function $f(x, y) = 2x - y$, sketch a contour diagram with at least 4 labeled contours. Describe in words how they are spaced.

2(15pts) Let $\vec{u} = \langle 1, -1, 1\rangle$, $\vec{v} = \langle 1, 0, 1\rangle$. Find the following.

(a) The angle between $\vec{u}$ and $\vec{v}$.

(b) A vector $\vec{w}$ perpendicular to both vectors.

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3(10pts) An object moves from point $P = (2, -1, 2)$ to $Q = (1, 0, 2)$, and a force, $\vec{F} = \langle 0, 1, 1 \rangle$, acts on the object. Find the projection of $\vec{F}$, $\vec{F}_{\text{parallel}}$, in the direction of $\vec{PQ}$.

4(10pts) For $\lim_{(x,y)\to(0,0)} \dfrac{x^2 - y^2}{x^2 + 2y}$, find the limit if it exists. If the limit does not exist, explain why not.

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5(15pts) (a) For function $w = f(x, y, z) = x^2 + xy - z$, find the directional derivative at $(1, 0, -5)$ in the direction of $\langle -1, 1, 0 \rangle$.

(b) Find the direction (in a unit vector) in which the function increases most rapidly at point $(1, 0, -5)$

6(12pts) Verify that the point $(2, 1, -1)$ is on the surface defined by the equation $x = y^2 - z^2 + 2$. Then viewing the surface as a level surface of a function $f(x, y, z)$ to find a vector normal to the surface.

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7(12pts) Find an equation of the tangent plane to the graph of the function $z = f(x, y) = \ln(x^2 + 1) + y$ at $(x, y) = (0, 2)$.

8(14pts) (a) For functions $z = f(x, y) = ye^x$, $x = u(s, t) = \ln s$, and $y = v(s, t) = \sin(s + t)$, use the chain rule to find $\dfrac{\partial z}{\partial s}$. (Simplification is not needed.)

(b) Find the value of $\dfrac{\partial z}{\partial s}$ at $(s, t) = (1, -1)$.

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2 Bonus Points: Calculus was discovered in: (a) the 16th century, (b) the 17th century, (c) the 18th century, (d) the 19th century.

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