Feb. 14, 2003 **MATH 208 Exam I** **Spring Semester, 2003**

Name:________________________ Score:__________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(15pts) (a) Let $z = f(x, y)$, $x = g(u, v)$, $y = h(u, v)$. If $f_x(1, 2) = 1$, $f_y(1, 2) = 2$; $g_u(0, 1) = 0$, $g_v(0, 1) = 1$; $h_u(0, 1) = 1$, $h_v(0, 1) = 0$, do you have enough information to evaluate $\frac{\partial z}{\partial u}|_{\{(u,v)=(0,1)\}}$? If not, what are the missing pieces of information?

(b) Let $w = xyz + x^2, x = s\sin t, y = t\ln s, z = st$, find $\frac{\partial w}{\partial s}|_{\{(s,t)=(1,0)\}}$. Also sketch a chain rule tree diagram for the problem.

2(20pts) For $z = f(x, y) = x^2 - y^3 + x^2y$, find the following

(a) The directional derivative $f_{\vec{u}}(1, 0)$ in the direction towards the point $(2, 1)$.

(b) The direction at which the function increases the most at $(1, 0)$.

(c) The maximum rate of change for the function at $(1, 0)$.

(d) An equation of the tangent line to the contour curve through $(1, 0)$.

3(15pts) Some values of a function $z = f(x, y)$ are given in the table

(a) Estimate $f_x(1, 2), f_y(1, 2)$.

(b) Estimate $f_{xy}(1, 2)$

$y \setminus x$	.975	1	1.25	1.5
1.75	.985	.9	.95	.95
2	1.12	1	.98	.975
2.25	1.25	1.2	1.1	1
2.5	1.3	1.25	1.2	1.05

4(15pts) Let $z = f(x, y) = e^{xy}\tan(x + y^2 - 1)$.

(a) Find the 1st order Taylor approximation $L(x, y)$ of the function at $(x, y) = (1, 0)$.

(b) What is $f(0.9, 0.1)$ approximately if you use the tangent plane approximation?

5(15pts) Consider the surface $x^2 + xz + y^2 + yz + z^2 = 3$ at the point $(1, 0, 1)$.

(a) Find a vector normal to the surface at the point $(1, 0, 1)$.

(b) Find an equation for the tangent plane to the surface at the point.

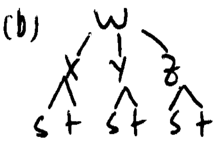
6(15pts) Some contour curves for a function $z = f(x, y)$ are sketched in the figure. Use the graph to determine the signs of $f_x, f_y, f_{xx}, f_{xy}, f_{yy}$ at the given point.

The End

Math 208 Test 1 Soln Key

1\. (a) No we don't. the missing information are $g(0,1)=1$, $h(0,1)=2$.

(6) (b)



$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s} = (yz+2x)\sin t + xz\frac{t}{s} + xy(t)$$

$= 0$ @ $s=1, t=0$.

2 (20) $z = f(x,y) = x^2 - y^3 + x^2y$. (a) $\nabla f(1,0) = (f_x(1,0), f_y(1,0)) = (2x+2xy, -3y^2+x^2)|_{(1,0)}$ $= (2,1)$. $\vec{v} = (2-1, 1-0) = (1,1)$, $\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$. $f_{\vec{u}}(1,0) = \nabla f(1,0)\cdot\vec{u} = \frac{3}{\sqrt{2}}$

(b) $\nabla f(1,0) = 2\vec{i} + \vec{j}$, or $\frac{\nabla f(1,0)}{\|\nabla f(1,0)\|} = \frac{2}{\sqrt{5}}\vec{i} + \frac{1}{\sqrt{5}}\vec{j}$ (c) $\|\nabla f(1,0)\| = \sqrt{5}$

(d) $\nabla f(1,0)\cdot(x-1, y-0) = 0 \Rightarrow 2(x-1)+y=0$ $2x+y=2$

3 (15) (a) $f_x(1,2) \doteq \frac{f(1.25,2)-f(1,2)}{0.25} = \frac{.98-1}{.25} = -\frac{0.02}{.25} = -0.08$. ~~$f_y(1,2)$~~

$f_y(1,2) \doteq \frac{f(1,2.25)-f(1,2)}{.25} = \frac{1.2-1}{.25} = \frac{.2}{.25} = 0.8$ (b) $f_x(1,2.25) \doteq$ ~~$f(1.25,2.25)$~~

$\doteq \frac{f(1.25,2.25)-f(1,2.25)}{.25} = \frac{1.1-1.2}{.25} = -\frac{.1}{.25} = -0.4 \Rightarrow f_{xy}(1,2) \doteq \frac{f_x(1,2.25)-f_x(1,2)}{.25}$

$= \frac{-0.4-(-0.08)}{.25} = \frac{-0.32}{.25} = -1.28$

4 (15) (a) $f_x(1,0) = ye^{xy}(\tan(x+y^2-1)) + e^{xy}\sec(x+y^2-1)|_{(1,0)} = 1$, $f_y(1,0) =$

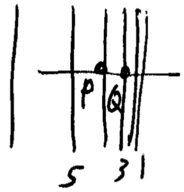
$= xe^{xy}\tan(x+y^2-1) + e^{xy}\sec(x+y^2-1)\cdot 2y|_{(1,0)} = 0 \Rightarrow L(x,y) = f(1,0) + f_x(1,0)(x-1)$

$+ f_y(1,0)(y-0) = 0 + 1(x-1) \Rightarrow L(x,y) = x-1$. $f(0.9, 0.1) \approx L(0.9, 0.1) = -0.1$

5 (15) $F(x,y,z) = x^2+xz+y^2+yz+z^2$. $F(1,0,1) = 3$. $(1,0,1)$ is on the level surface $F(x,y,z)=3$. (a) $\nabla F(1,0,1) = (F_x, F_y, F_z)|_{(1,0,1)} = (2x+z, 2y+z,$ $x+y+2z)|_{(1,0,1)} = (3,1,3)$. (b) $\nabla F(1,0,1)\cdot(x-1, y-0, z-1) = 0$

$3(x-1)+y+3(z-1)=0 \Rightarrow 3x+y+3z=6$

6 (15)



$f_y(P) = f_{xy}(P) = f_{yy}(P) = 0$ since there are no changes in y. $f_x(P) < 0$ since f decreases w/ increase in x. $f_{xx}(P) < 0$ since the rate f_x at which f decreases also decreases. in fact, $f_x(Q) < f_x(P)$.

End.