May 5, 2003 | # MATH 208 Final Exam | **Spring Semester, 2003**

Name:________________________ Section:________________ Instructor:______________

Instructions:

(1) You must show supporting work to receive credits.
(2) Use exact values whenever possible, e.g. $\pi/4$ instead of 0.785398...
(3) Calculators are allowed.
(4) Make sure you have all the 7 pages of questions.

Problem	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Point	14	16	16	16	16	18	14	18	12	16	14	14	16	200
Credit														

1. (14 points) Let $f(x, y) = \sqrt{4x + 5y}$.

   (a) (10 points) Find the quadratic approximation to $f(x, y)$ at the point $(4, -3)$.

   (b) (4 points) Use the quadratic approximation of (a) to approximate $f(4.1, -2.9)$.

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2. (16 points) Let S be the surface given by $\dfrac{6-z^2}{x^2+y^2}=1$.

   (a) (4 points) Verify that point $P=(1,1,2)$ is on the surface.

   (b) (6 points) Find a normal vector of S at the point P.

   (c) (6 points) Write down an equation of the tangent plane to S at P.

3. (16 points) Use polar coordinates to evaluate the following integral:

$$\int_0^2 \int_0^{\sqrt{4-y^2}} e^{x^2+y^2}\,dx\,dy.$$

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4. (16 points) Let $f(x, y) = 3xy^2 + 12xy - 4x^2$.

   (a) (8 points) Find all critical points of f.

   (b) (8 points) Classify each critical point of f as a local maximum, a local minimum or a saddle point.

5. (16 points) Use the method of Lagrange Multipliers to find the point on the plane $4x - 3y + 5z = 48$ closest to the point $(0, 4, -3)$. (*Hint:* Minimize the square of the distance from $(0, 4, -3)$ to (x, y, z).)

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6. (18 points) Let $f(x, y, z) = 2x^2 \ln(y) + xe^{2z}$, let $P = (-2, 1, 0)$ and $Q = (2, 5, -2)$.

   (a) (6 points) Find and simplify $\bigtriangledown f(-2, 1, 0)$.

   (b) (6 points) Find and simplify the rate of change of f at P in the direction from P to Q.

   (c) (6 points) Find and simplify the unit vector in the direction in which f decreases most rapidly at P.

7. (14 points) Consider $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (x+z)\,dz\,dy\,dx$. Change the integral to an iterated triple integral in the spherical coordinates. (**DO NOT EVALUATE the triple integral.**)

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8. (18 points) Let $\vec{F}(x, y) = (x^2 + 2xy)\vec{i} + (x^2 + 5y + 1)\vec{j}$.

   (a) (6 points) Use a derivative test to verify that the vector field $\vec{F}(x, y)$ is a conservative vector field.

   (b) (8 points) Find a potential function f for $\vec{F}(x, y)$.

   (c) (4 points) Find the $\int_C \vec{F}(x, y) \cdot d\vec{r}$, where C is a smooth path from $(1, 0)$ to $(0, 1)$.

9. (12 points) Consider the integral $\int_0^2 \int_{x^4}^{8x} f(x, y)dydx$.

   (a) (4 points) Sketch or describe the region of integration.

   (b) (8 points) Switch the order of integration.

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10. (16 points) Let $\vec{F}(x, y) = -y\vec{i} + xj$ and C be the line from $(1, 0)$ to $(2, 3)$.

    (a) (6 points) Find a parameterization of C.

    (b) (10 points) Find the line integral $\int_C \vec{F} \cdot d\vec{r}$.

11. (14 points) Let S be the portion of the paraboloid $z = 4 - x^2 - y^2$ lying above the xy-plane, with upward orientation. Find the flux $\int_S \vec{F} \cdot d\vec{A}$ through S, where the vector field is $\vec{F}(x, y, z) = x\vec{i} + y\vec{j} + z\vec{k}$.

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12. (14 points) Suppose $\vec{F}$ is a smooth vector field defined everywhere such that $\text{div}\vec{F} = 10$. Find the flux of $\vec{F}$ out of a closed cylinder (with cover and base) of height 5 and radius 2, centered on the z-axis with base in the xy-plane.

13. (16 points) Let $\vec{F} = (y + 2z)\vec{i} + 4x\vec{j} + yz\vec{k}$.

    (a) (6 points) Find the curl of $\vec{F}$: $\nabla \times \vec{F}$.

    (b) (10 points) Use your result from part (a) to find the line integral around the circle of radius 1 in the xy-plane, centered at the origin, oriented counterclockwise when viewed from above.