Math 208 Sec. 1 **Test 3** **April 20, 2001**

Name:____________________________________ Score:____________

Instructions: You must show supporting works to receive full and partial credits.

1(15 pts) Let $\vec{v} = 2\vec{i} - \vec{j} + 3\vec{k}$ be a constant force field.

(a) Find the work down by $\vec{v}$ along the line from point $(1, 1)$ to $(-1, 0)$.

(b) Find the flux of $\vec{v}$ through the rectangular plate as shown.

2(15 pts) Consider the vector field $\vec{F}(x, y) = x\vec{i} + y\vec{j} + (x^2 + y + 2z^2)\vec{k}$.

(a) Find the divergence, div$\vec{F}$, at point $(1, 0, 1)$.

(b) Find the curl, curl$\vec{F}$, at the same point $(1, 0, 1)$.

3(15 pts) Set up a definite integral in one-variable for the line integral $\int_C \vec{F} \cdot d\vec{r}$ of $\vec{F} = x\vec{i} + \vec{j} + \vec{k}$ along the line C from $(1, 0, 0)$ to $(2, 3, -1)$. Note: **Do Not** evaluate the integral.

4(20 pts) (a) Verify that the vector field $\vec{F}(x, y) = (2x+y^3)\vec{i}+(3xy^2-1)\vec{j}$ is a conservative vector field.

(b) Find a potential function f for $\vec{F}(x, y)$.

(c) Find the work done by $\vec{F}(x, y)$ from $(1, 0)$ to $(-1, 0)$ along the upper half circle $x^2 + y^2 = 1, y \geq 0$.

5(15 pts) Use Green's Theorem to find the circulation of $\vec{F}(x, y) = (\sin x^2 + 2y)\vec{i} + (y^2 + x^2)\vec{j}$ around the square with corners $(0, 0), (1, 0), (1, 2), (0, 2)$ with the counterclockwise orientation.

6(20 pts) Consider the vector field $\vec{F} = (y + 2x)\vec{i} + (z + y)\vec{j} - x\vec{k}$.

(a) **Set up** a double integral for the flux of $\vec{F}$ through the surface $S : z = x^2+y$ which is over the square $0 \leq x \leq 1, 1 \leq y \leq 3$ on the xy-plane and oriented upward. Simplify but **DO NOT** evaluate the integral.

(b) Use the Divergence Theorem to find the flux of the same vector field $\vec{F}$ through the surface of the solid cube as shown.

Bonus(6 pts) Consider the vector field $\vec{F}(x, y, z) = -y\vec{i} + x\vec{j} + (z^2 + x)\vec{k}$.

(a) If $\mathcal{S}$ is a closed surface lying above the xy-plane, what would be the sign for the flux of $\vec{F}$ through $\mathcal{S}$ assuming outward orientation for $\mathcal{S}$? Explain.

(b) Is it possible for the circulation density of $\vec{F}$ (at any point and around any direction) be 3 per unit area? Explain. (Hint: Find the curl of $\vec{F}$ first.)

The END