Math 208 **Test I** **February 9, 2001**

Name:_____ Score:_____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(20 pts) (a) Find $\dfrac{\partial H}{\partial t}(1,0)$ if $H(p,t) = \ln(e^{tp}+t) + \sin(p^2 t)$.

(b) If $w = x^3 + y^2 + z, x = rs, y = r^2 - s^2, z = r + s$, find $\dfrac{\partial w}{\partial r}$. (No need to simplify your answer.)

2(15 pts) (a) Find an equation of the tangent plane to the surface $x^3 - y^2 - z = 1$ at the point$(x, y, z) = (2, 3, -2)$.

(b) Find a normal vector to the surface.

3(15 pts) The volume of a cylinder with radius r and height h is given by $V = \pi r^2 h$. Use differential to find the approximate maximum error $\triangle V$ if the measured radius and height are $r = 1$ foot and $y = 2$ feet with maximum errors of 0.01 feet each.

4(18 pts) (a) If $f(3,2) = 1.1$ and $f(3, 2.2) = 1$, approximate f_y at $(3,2)$.

(b) Find the direction derivative of $z = f(x,y) = x^2 + y$ at point $(0,1)$ in the direction of $3\vec{i} - 4\vec{j}$.

5(16 pts) The value and all partial derivatives up to the 2nd order of a function $z = f(x,y)$ at $(1,2)$ are given as: $f = 1, f_x = -2, f_y = 1, f_{xx} = 0, f_{xy} = 0.5, f_{yy} = -1$.

(a) Find the gradient at $(1,2)$.

(b) Find the minimum rate of change at $(1,2)$ and the direction at which the minimum rate occurs.

(c) Find the 2nd order (quadratic) Taylor approximation of f near $(1,2)$ and use it to approximate the value of f at $(1.1, 1.9)$.

6(16 pts) Some level curves for a function $z = f(x,y)$ are sketched in the figure. Use the graph to determine the signs of f_x, f_{xx}, f_{xy} at the dotted point.

END