Math 208 Fall 2018 **Exam 1**

Print Your Name Legibly: ____________________ **Score:** __________

Instructions: You must show supporting work to receive full and partial credits. No textbook, notes, cheat sheets, calculators allowed.

1(12pts) Given three points $P = (1, 0, 2)$, $Q = (0, 1, 2)$, $R = (1, 1, 1)$, find the following.

(a) The displacement vectors $\vec{PQ}$ and $\vec{PR}$.

(b) A vector perpendicular to the plane containing these points.

2(12pts) For two vectors $\vec{u} = \langle 1, 0, 1\rangle$, $\vec{v} = \langle 1, -1, 0\rangle$,

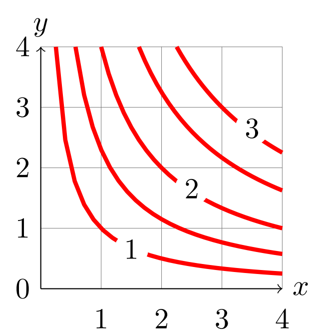
(a) Find the angle between them.

(b) Find the projection of $\vec{u}$, $\vec{u}_{\text{parallel}}$, in the direction of $\vec{v}$.

3(10pts) For $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x+y^2}{2x+y^2}$, find the limit if it exists. If the limit does not exist, explain why not.

4(15pts) (a) For function $w = f(x,y,z) = x^2 + 2xy - z$, find its directional derivative at $(1,0,-5)$ in the direction of $2\vec{i} - 2\vec{j} + \vec{k}$.

(b) The figure below shows the level curves of a two-variable function $z = f(x,y)$. At the point $(1,1)$ draw a vector representing the gradient $\nabla f(1,1)$. Explain how you know the direction and the length of the vector.



5(12pts) Find an equation of the tangent plane to the surface defined by the equation $x + y^2 + z^3 = 2$ at point $(2,1,-1)$.

6(12pts) For function $f(x,y) = \dfrac{xy}{1+y}$ find $f_{xy}(x,y)$.

7(12pts) For a function $z = f(x,y)$, you are given its partial derivatives:

$$\frac{\partial f(x,y)}{\partial x} = \frac{x}{\sqrt{x^2+y^2}}, \qquad \frac{\partial f(x,y)}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}.$$

Find $\dfrac{\partial z}{\partial u}$ if in addition $x = g(u,v) = u+v,\ \ y = h(u,v) = 2uv$. (Simplification is not needed.)

8(15pts) For function $z = f(x,y) = xe^{xy}$.

(a) Find the Taylor polynomial of degree 1 for $f(x,y)$ near $(1,0)$.

(b) You are given these values $f_{xx}(1,0) = 0$, $f_{xy}(1,0) = 2$, $f_{yy}(1,0) = 1$, find the Taylor polynomial of degree 2 for $f(x,y)$ near $(1,0)$.

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2 Bonus Points: Calculus was discovered in: (a) the 16th century, (b) the 17th century, (c) the 18th century, (d) the 19th century.