Math 208 Fall 2012 **Exam 5**

Name: ______________________ **Score:** ______________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(16pts) (a) The curve C is the intersection of the cylinder $x^2 + y^2 = 4$ with the plane $z = y$, starting at the point $(-2, 0, 0)$ and moving counter-clockwise to the point $(2, 0, 0)$. Write a parameterization of the curve.

(b) Find a parameterization of the plane through point $P(1, 0, 1)$, $Q(2, 1, 3)$ and parallel to the vector $\vec{v} = \langle 1, 2, 3 \rangle$.

2(16pts) Let C be the counterclockwise closed path connecting the points $(0, 0)$, $(1, 0)$, $(0, 2)$ in line segments. Use Green's Theorem to find the circulation of the vector field $\vec{F} = \langle x^2 - y,\ y^2 + 2x \rangle$ along C.

3(16pts) (a) Find the work done by the force field $\vec{F} = \langle 3, 2, 1 \rangle$ on an object moving on the line from the origin to point $P(1, 1, 1)$.

(b) Let $z = f(x, y) = \arctan(xy) + x$ and $\vec{F} = \nabla f(x, y) = \langle 1 + y/(1 + x^2y^2),\ x/(1 + x^2y^2) \rangle$. Use the fundamental theorem of line integrals to calculate the line integral of $\vec{F}$ along the oriented path $C : y = x^3, -1 \leq x \leq 1$.

4(16pts) Let C be the helix $\vec{r}(t) = \langle 2\cos t,\ 2\sin t,\ t \rangle$ with $0 \leq t \leq 2\pi$ and $\vec{F} = \langle -y,\ x,\ z \rangle$. Find the line integral $\int_C \vec{F} \cdot d\vec{r}$.

5(16pts) (a) Use the curl test to verify that the vector field $\vec{F} = \langle 2ye^{2x},\ 2y + e^{2x} \rangle$ is conservative.

(b) Find a potential function $z = f(x, y)$ for the vector field.

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2 Bonus Points: True or False: Nebraska has three electoral college votes.