Name:
Score:
Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.
$\mathbf{1}(\mathbf{1 6 p t s})$ Find all critical points of $z=f(x, y)=4 x y-x^{4}-\frac{1}{2} y^{2}$, and use the 2 nd derivative test to classify them as local minimum, local maximum, saddle, or none of the above.
$\mathbf{2 ( 8 p t s )}$ Some level curves of a function $z=f(x, y)$ and a curve $g(x, y)=c$ are sketched in the figure below.

(a) Locate approximately and label all points at which $\nabla f(x, y)=\lambda \nabla g(x, y)$.
(b) Locate approximately and label the constraint maximum point and the constraint minimum point for $f$ subject to $g(x, y)=c$.
$\mathbf{3}(\mathbf{1 6 p t s})$ Consider an iterated double integral $\int_{0}^{4} \int_{\sqrt{y}}^{2} \sqrt{1+x^{3}} d x d y$.
(a) Sketch the region of the integral.
(b) Switch the order of integral.
(c) Evaluate the integral from (b).
$4(16 \mathrm{pts})$ A solid $G$ is bounded by these surfaces: $x=0, x=2, y=0, z=0$, and $3 y+z=3$, as shown.
(a) Set up the iterated integral in the order of $d z d y d x$ for a triple integral $\int_{G} f(x, y, z) d V$.

(b) Set up the iterated integral in the order of $d x d y d z$ for a triple integral $\int_{G} f(x, y, z) d V$.
$\mathbf{5}$ (8pts) Some level curves of a function $z=f(x, y)$ are as shown, and consider the double integral $I=\int_{1}^{4} \int_{3}^{5} f(x, y) d y d x$. Find an underestimate for the integral with $\Delta x=1$ and $\Delta y=2$.

$\mathbf{6}(\mathbf{1 6 p t s})$ Use the Lagrange multiplier method to find the maximum and minimum of the function $w=f(x, y, z)=x+2 y+3 z$ with $(x, y, z)$ satisfies $x^{2}+y^{2}+z^{2}=14$.

