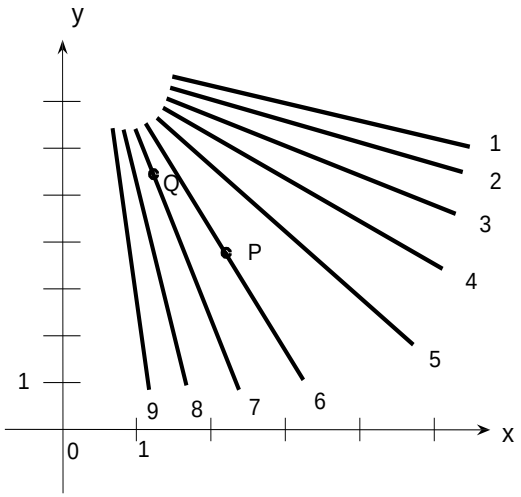
Math 208 Fall 2012 **Exam 2**

Name: _____ **Score:** _____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(20pts) A contour diagram for a smooth function $z = f(x, y)$ is sketched in the figure below.



(a) Determine the sign of $f_x(P)$ and $f_y(P)$.

(b) Which value is greater, $|f_x(P)|$ or $|f_y(P)|$, and why?

(c) Sketch the direction of grad$f(P)$ and grad$f(Q)$.

(d) Which vector is longer, grad$f(P)$ or grad$f(Q)$, and why?

(e) What is the sign of $f_{\vec{u}}(P)$, where $\vec{u}$ is the unit vector from P to Q, and why?

(f) Is $f_{xx}(P)$ positive or negative, and why?

(g) Is $f_{xy}(P)$ positive or negative, and why?

(*Page 1. Continue on Next Page ...*)

2(16pts) A surface is given by the equation $x^2 + xyz = 2$.

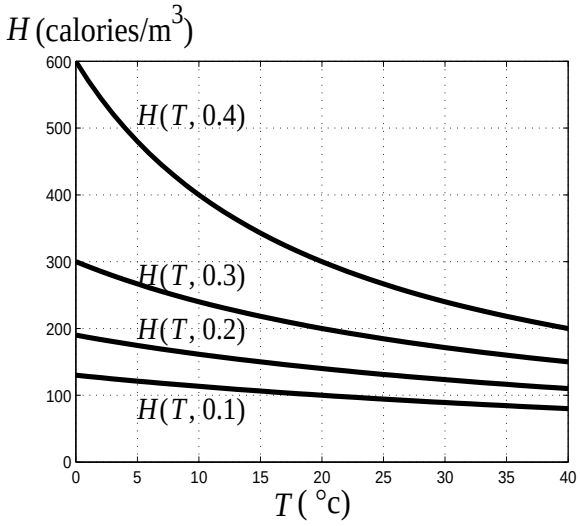
(a) Verify that point $(2, -1, 1)$ is on the surface.

(b) Find a vector $\vec{n}$ that is normal to the surface at the point.

(c) Find an equation for the tangent plane to the surface at the point.

3(16pts) An airport can be cleared of fog by heating the air. The amount of heat required depends on the air temperature and the wetness of the fog. The heat $H(T, w)$ required (in calories per cubit meter of fog) as a function of temperature T (in degrees Celsius) and the water content w (in grams per cubic meter of fog). The figure below shows some section curves of H.

(a) Show work to estimate $H_T(20, 0.3)$ with unit.



(b) Show work to estimate $H_w(20, 0.3)$ with unit.

(*Page 2. Continue on Next Page ...*)

4(18pts) Let $z = f(x,y) = \dfrac{4}{x+2y}$.

(a) Find the linear, $L(x,y)$, Taylor polynomial near $(2,1)$.

(b) Find the quadratic, $Q(x,y)$, Taylor polynomial near $(2,1)$.

(c) Which approximation, $L(1.9,1)$ or $Q(1.9,1)$, is closer to the exact value of the function $f(1.9,1)$? Show the values.

5(10pts) A function $f(x,y)$ is *homogeneous of degree* p if $f(tx,ty) = t^p f(x,y)$ for all x, y and all t.

(a) Let $g(t) = f(tx,ty)$, find $g'(1)$.

(b) Let $h(t) = t^p f(x,y)$, find $h'(1)$.

(c) Show that every homogeneous of degree p function $z = f(x,y)$ satisfies the identity: $xf_x(x,y) + yf_y(x,y) = pf(x,y)$.

---

2 Bonus Points: Physics is build on: (a) philosophy, (b) chemistry, (c) history, (d) calculus.