Math 208 **Test 3** **Dec. 7, 2001**

Name:____________________________________ Score:_____________

Instructions: You must show supporting works to receive full and partial credits.

1(15 pts) (a) Find the work done by the force field $\vec{F} = (y - x + 3)\vec{i} + 4\vec{j} - \vec{k}$ from $(0, 0, 0)$ to $(1, 1, 1)$.

(b) Find the flux of $\vec{v} = 3\vec{i} + \vec{j} - 5\vec{k}$ through any disc of radius 2 which is parallel to the xy-plane.

2(15 pts) (a) Find the curl, curl$\vec{F}$, of vector field $\vec{F}(x, y) = (z + y)\vec{i} + (z^2 + x^2)\vec{j} + (x^3 + y^3)\vec{k}$ at point $(1, 0, 1)$.

(b) Find the circulation density $\text{circ}_{\vec{n}}\vec{F}(1, 0, 1)$ of the same $\vec{F}$ as in (b), at the same point $(1, 0, 1)$, and in the direction of $\vec{a} = (1, 0, 1)$.

3(15 pts) If the curl of a vector field $\vec{F}$ is a constant vector curl$\vec{F}(x, y, z) = \vec{i} + \vec{j} + \vec{k}$, find the circulation of $\vec{F}$ around the square as shown.

4(20 pts) (a) Show that the vector field $\vec{F}(x, y) = (2x + y^3 + z)\vec{i} + (3xy^2 - 1 + z)\vec{j} + y\vec{k}$ is not path independent.

(b) Find the line integral $\int_{(1,2)}^{(0,1)} (2x + y)dx + (x - 1)dy$ using the fundamental theorem of line integral.

5(15 pts) Use Green's Theorem to find the circulation of $\vec{F}(x, y) = (\sin x^2 + 2y)\vec{i} + (\cos y^2 + x)\vec{j}$ around the circle $x^2 + y^2 = 1$ tracing counterclockwise.

6(20 pts) (a) **Set up** a double integral for the flux of vector field $\vec{F}(x, y, z) = x\vec{i} + y\vec{j} + z\vec{k}$ through the part of a parabola $S : z = x^2 + y^2$ over the square $0 \le x \le 1, 1 \le y \le 3$, assuming S is oriented upward. Simplify but **DO NOT** evaluate the integral.

(b) Use the Divergence Theorem to find the flux of the same vector field $\vec{F}$ through the surface of any solid sphere of radius R.

Bonus(3 pts) Circle your bet on Husker's Women Volleyball match against Colorado State tonight at Coliseum: Win Lose

The END