Name:_____   Score:_____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(15 pts) (a) Find $\dfrac{\partial G}{\partial x}(1,0)$ if $G(x,y) = \ln(x+y^2) + x + e^{x^2 y}$.

(b) If $w = x^2 z + y^3 x, x = t^2 + s^2, y = st, z = \cos(t)$, use a chain rule to find $\dfrac{\partial w}{\partial s}$. (No need to simplify your answer.)

2(15 pts) (a) Find an equation of the tangent plane to the surface $z = f(x,y) = x^3 - y^2 + 1$ at the point $(x,y,z) = (1,2,-2)$.

(b) If $g(1,2,1) = 7, g_x(1,2,1) = 3, g_y(1,2,1) = -1, g_z(1,2,1) = 2$, find the unit vector in whose direction function $w = g(x,y,z)$ changes most rapidly.

3(10 pts) The area of a triangular with base x and height y is given by $A = xy/2$. Use differential to find the approximate maximum error $\triangle A$ if the measured values are $x = 2, y = 2$ feet with a maximum error of 0.02 feet each.

(**Bonus 2 pts**) What is the maximum relative error for the area?

4(15 pts) (a) Find the directional derivative of $z = f(x,y) = x^2 + y$ at point $(1,-1)$ in the direction from $(1,-1)$ to $(5,2)$.

(b) Find an equation of the tangent line at $(1,-1)$ to the level curve of f through the point $(-1,1)$.

(**Bonus 2 pts**) If you move the point $(1,-1)$ to another point which is $1/2$ unit distance away $(1,-1)$, in the direction of the gradient, what would be an approximating value of change in f between these two point?

5(15 pts) The value and all partial derivatives up to the 2nd order of a function $z = f(x,y)$ at $(1,2)$ are given as: $f = 1, f_x = -2, f_y = 0, f_{xx} = 4, f_{xy} = 0.5, f_{yy} = -1$.

(a) Use the tangent plane approximation to estimate the value $f(1.1, 1.98)$.

(b) Use the Taylor polynomial of degree 2 to approximate the same value $f(1.1, 1.98)$.

(**Bonus 2 pts**) Which value, 0.78 or 0.83, is more likely to be the true value of $f(1.1, 1.98)$ and why?

6(15 pts) Some level curves for a function $z = f(x,y)$ are sketched in the figure.

(a) Use difference ratio to estimate the value of f_x at the dotted point.

(b) Determine the signs of f_y, f_{yx} at the dotted point.

7(15 pts) (a) Find an equation of a plane that contains these three points $P = (1,0,1), Q = (1,2,2), R = (3,0,-1)$.

(b) Find the area of the triangle $\triangle PQR$.

(**Bonus 2 pts**) Find the distance from point R to the line through P and Q.

END