

Math 208 Summer II 2005 Exam 2

August 3, 2005

NAME _____

Show all work; guessing is not enough. 50 points possible (counting for 100 points in the course).

1. (5 pts) Find all second partial derivatives of $f(x, y) = x^2y^3 + e^{2x} \sin y$

2. Let $z = f(x(t), y(t))$ be a composition of functions.
 - (a) (5 pts) Express $\frac{dz}{dt}$ using the chain rule.

 - (b) (5 pts) If $z = 1 - x^2 - y^2$, where $x = \cos t$ and $y = 1 + \sin t$, then use part (a) to determine $\frac{dz}{dt}$.

3. (a) (5 pts) Find the magnitude of the steepest slope on the surface $z = x^2 + 2y$ at the point $(3, 2, 11)$.

(b) (5 pts) For the same surface, find the directional derivative in the direction $\langle x, y \rangle = \langle \frac{4}{5}, \frac{3}{5} \rangle$.

4. (5 pts) Find the linear approximation of $z = f(x, y) = x^2 + y^2$ at $(x, y) = (2, 3)$. Check how close the approximation is at $(x, y) = (2.01, 3.02)$.

5. (6 pts) A cardboard box for mailing is x by x by y inches. Use Lagrange multipliers to find its maximum volume if the Postal Service requires that length plus girth ($= y + 4x$) not exceed 84 inches.

6. (7 pts) Find the critical point of $f(x, y) = x^2 - xy + y$, and use the second derivative test to identify whether it is a local maximum, local minimum, or saddle point.

7. (7 pts) Determine the volume of the solid region bounded by the surfaces $z = 4 - y^2$, $x = y$, and the coordinate planes.