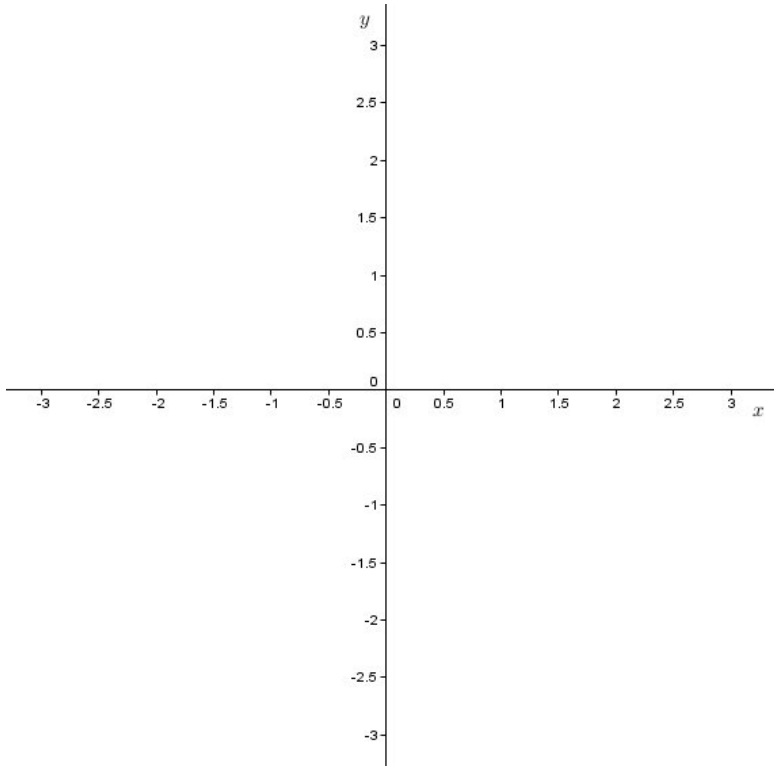
NAME:______________________________

Professor's name:______________________ Lecture start time:_____________________

This exam should have 10 pages; please check that it does. Show all work that you want considered for grading. Calculators are not allowed. **An answer will only be counted if it is supported by all the work necessary to get that answer.** Simplify as much as possible, except as noted: for example, don't write $\cos(\pi/4)$ when you can write $\sqrt{2}/2$. Also, **give exact answers only**, except as noted; for example, don't write 3.1415 for π. If there are multiple parts to a problem, the parts are not necessarily worth an equal amount of points.

Problem	**Maximum**	
1	7	
2	12	
3	12	
4	13	
5	16	
6	14	
7	10	
8	15	
9	10	
10	12	
11	10	
12	10	
13	17	
14	10	
15	15	
16	17	
Total	200	

1. (7 points) Let $z = f(x,y) = -x^2 - y^2$. Draw level curves for $z = -1, -2, -3$ and -4 on the axes below, making sure to label each level curve.



2. (12 points). Suppose $z = f(x,y)$, $x = x(t,s)$ and $y = y(t,s)$.

   (a) Write down the chain rule for $\dfrac{\partial z}{\partial s}$.

   (b) Find $\dfrac{\partial z}{\partial s}$ when $t = 0$ and $s = 6$ if

$$f_x(3,2) = 1, \quad f_y(3,2) = 5, \quad f_x(0,6) = -2, \quad f_y(0,6) = 4$$

$$x(0,6) = 3, \quad y(0,6) = 2, \quad x_t(0,6) = -4, \quad x_s(0,6) = 10, \quad y_t(0,6) = -2, \quad y_s(0,6) = -3.$$

3. (12 points) Consider the plane $5x - y + 7z = 21$.

   (a) Find a point on the x-axis on this plane.

   (b) Find a vector perpendicular to the plane.

   (c) Is the vector $\vec{i} - 2\vec{j} - \vec{k}$ parallel to the plane? Explain your answer.

4. (13 points) Let $f(x, y) = e^{x^2+y}$.

   (a) Compute grad f.

   (b) Find $f_{\vec{u}}(1, -1)$, which is the directional derivative of f in the direction of $\vec{i} + 2\vec{j}$ at $(1, -1)$.

   (c) In what direction is the maximum directional derivative of f at the point $(1, -1)$? What is that maximum directional derivative?

5. (16 points) Find the critical points of $f(x, y) = xy + x^2 + (y^3/3)$, and classify each of them as a local maximum, local minimum or saddle point.

6. (14 points) Using Lagrange multipliers, determine the shortest distance from the line $x + y = 9$ to the point $(0, 1)$. Hint: minimize the square of the distance from (x, y) to $(0, 1)$. No credit will be given for any method besides Lagrange multipliers.

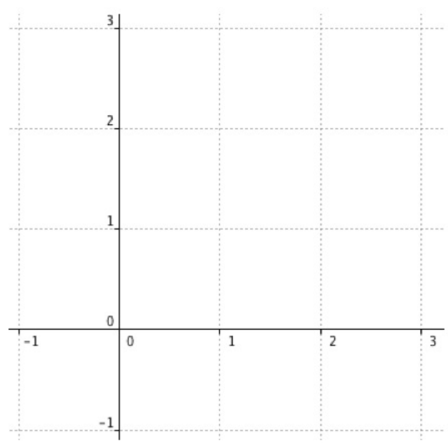
7. (10 points)

   (a) Find a normal vector to the surface $x^3 + 2yz = -5$ at $(1, -1, 3)$.

   (b) Find an equation for the tangent plane to the surface $x^3 + 2yz = -5$ at $(1, -1, 3)$.

8. (15 points) Consider the integral $\displaystyle\int_0^2 \int_y^2 \frac{ye^{x^2}}{x} dx\, dy$.

   (a) Sketch the region.



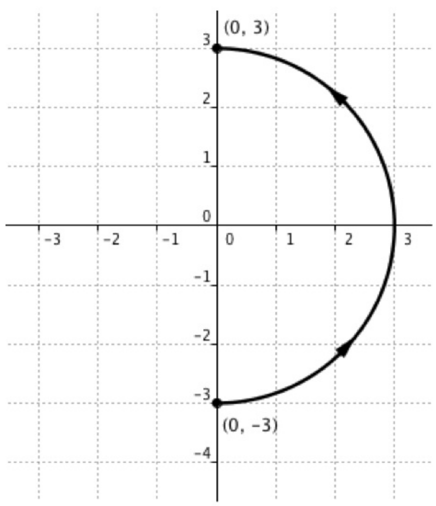
   (b) Change the order of integration to $dy\, dx$, and evaluate this integral.

9. (10 points) Let S be the closed cylindrical surface described as follows: The sides of the cylinder are $x^2 + y^2 = 4$ with $0 \le z \le 3$, and the cylinder is closed off by a top disk in the plane $z = 3$ and a bottom disk in the plane $z = 0$. Use the divergence theorem to calculate the outward flux of the vector field

$$\vec{F} = e^{y^2} \cos z\, \vec{i} + 2y\, \vec{j} - x^3 y\, \vec{k}$$

through S.

10. (12 points) Let $\vec{F} = x\vec{i} + z\vec{j} - y\vec{k}$. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where C is the half circle of radius 3 in the yz plane, centered at the origin, oriented counterclockwise when viewed from the positive x-axis, and going from $(0, 0, -3)$ to $(0, 0, 3)$ - see the picture, which is in the y-z plane.



11. (10 points) Let W be the "ice-cream cone" solid above the cone $z = \sqrt{x^2 + y^2}$ and below $x^2 + y^2 + z^2 = 9$. Its density is given by $\delta(z) = z$, where the units for the solid are centimeters cubed, and the density is in grams per centimeter cubed. Set up a triple integral in **spherical coordinates** that gives the mass of the cone. **DO NOT EVALUATE**.

12. (10 points) Let the position of a particle be given by

$$x(t) = (1/4)t^4 - (1/2)t^2 \qquad \text{and} \qquad y(t) = 3t^2 - 6t,$$

where x and y are in centimeters and t is in seconds.

(a) Find the velocity of the particle. Give the units.

(b) Find any times when the particle stops.

13. (17 pts) Use Stokes' theorem to calculate the circulation of the vector field $\vec{F} = y\,\vec{i} + 2x\,\vec{k}$ around the rectangle that goes from $(0,0,0)$ to $(2,0,0)$ to $(2,0,2)$ to $(0,0,2)$, and back to $(0,0,0)$.

14. (10 points) Let $\vec{F} = (xy)\vec{i} + (x^2 + xy)\vec{j}$. Consider the line integral $\int_C \vec{F} \cdot d\vec{r}$, where C is the square with vertices $(0,0)$, $(3,0)$, $(3,2)$ and $(0,2)$, oriented counterclockwise. Use Green's theorem to evaluate this integral.

15. (15 points) Let $\vec{F} = (3x^2y + 2)\vec{\imath} + (x^3 - 1)\vec{\jmath}$.

    (a) Find a potential function for $\vec{F}$.

    (b) Use the potential function to evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is part of the circle of radius 2 from $(2, 0)$ to $(0, 2)$ (that is, the part of the circle in the first quadrant.)

16. (17 points) Let S be part of $z = 4 - x^2 - y^2$ which is above the xy plane, oriented with upwards normals, and let $\vec{F} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$.

    Set up an iterated integral in polar coordinates for the flux of $\vec{F}$ over S. Simplify the integrand as much as possible, making sure that the variables in the integrand match the variables of integration, but **DO NOT EVALUATE**.