Name: ____________________

Circle the name of your instructor:

Brown 11:30 am	Deng 12:30 pm	DeVries 6:30 pm	Lindo 10:30 am	Lutz 9:30 am	Nasseh 12:30 pm
Pei 12:30 pm	True 8:30 am	True 1:30 pm	Webb 1:30 pm	Yang 1:30 pm	

Instructions

- There are **12** questions on **8** pages (including this cover sheet).
- **Turn off all communication devices.**
- No books or other notes are allowed.
- You may use a calculator, but an answer will only be counted if it is supported by all the work necessary to obtain that answer.
- Simplify as much as possible, except as noted. For example, write $\sqrt{2}/2$ instead of $\cos(\pi/4)$.
- Give exact answers only, except as noted. For example, use π instead of 3.1415 if π is the answer.
- **Show all your work** and explain your answers. Unsupported answers will receive **little credit**.
- If specified, use the method required by each problem. Alternate methods will not receive full credit.
- You have **2 hours** to complete the exam.

Good luck !

Question	Out of	Score
1	25	
2	25	
3	10	
4	15	
5	15	
6	15	
7	15	
8	15	
9	20	
10	15	
11	15	
12	15	
Total	200	

1(25pts) Let $\vec{v} = 2\vec{i} - 2\vec{j} + \vec{k}$ and $\vec{w} = 3\vec{i} - 4\vec{k}$.

(a) Find $\|\vec{v}\|$, $\|\vec{w}\|$.

(b) Find the angle between $\vec{v}$ and $\vec{w}$.

(c) Find a vector perpendicular to both $\vec{v}$ and $\vec{w}$.

(d) Find the area of the parallogram whose sides are formed by $\vec{v}$ and $\vec{w}$.

(e) Find the equation of the plane containing point $(1, 3, 2)$ and perpendicular to $\vec{v}$.

2(25pts) Let $f(x,y) = x + y^2 + x^2y$.

(a) Find $f_x(x,y)$, $f_y(x,y)$, $f_{xy}(x,y)$ and evaluate them all at the point $(1,2)$.

(b) Find the directional derivative of the function $f(x,y)$ at point $(1,2)$ in the direction of $\vec{v} = -3\vec{i} + 4\vec{j}$.

(c) Find the direction from the point $(1,2)$ in which $f(x,y)$ is increasing most rapidly.

(d) Find the maximal rate of increase of the function at the point $(1,2)$.

(e) Find the equation of the tangent plane of the graph $z = f(x,y)$ at the point $(1,2)$.

3(10pts) Let $f(x,y) = 3x^2y$ and $x(u,v) = u - v^2, y(u,v) = \frac{2u}{3v+1}$. Find $\frac{\partial f}{\partial v}$ in u, v, but do not simplify the expression.

4(15pts) Find the potential function for the vector field $\vec{F} = \langle 2xy, x^2 + 2 \rangle$ or show that it fails to exist.

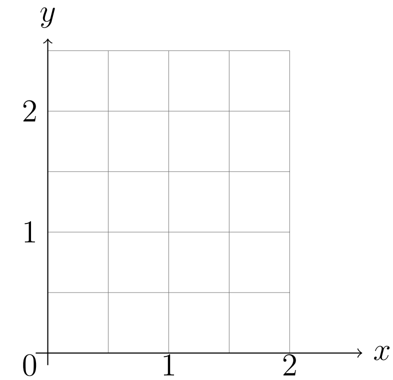
5(15pts) For the function $f(x,y) = y^2 + y + x^2 y - \dfrac{2}{3}x^3$,

(a) Verify that $(0, -\frac{1}{2})$ is a critical point. (You do not need to solve for all critical points.)

(b) Use the second derivative test to determine if $(0, -\frac{1}{2})$ is a local maximum, a local minimum, or a saddle point.

6(15pts) Use the Lagrange multiplier method to find the minimal value of the function $f(x,y) = x + 2y$ that is subject to the constraint $x^2 + 2y^2 = 3$.

7(15pts) For the iterated double integral $\int_0^2 \int_{y/2}^1 e^{x^2} \, dx dy$.

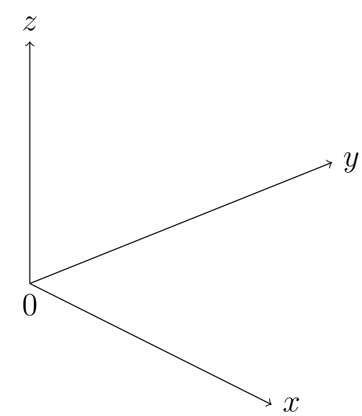


(a) Sketch the region of the integral.

(b) Change the order of integration to $dydx$. **Do not evaluate the integral.**

8(15pts) Let W be the solid bounded by the plane $6x + 3y + 2z = 6$ in the first octant.

(a) Sketch the solid.



(b) Set up an iterated triple integral in the order of $dxdydz$ for $\iiint_W z dV$. **Do not evaluate the iterated integral.**

9(20pts) Suppose the motion of a particle is given by $x = 4\cos t, y = \sin t, z = 2t$ for $0 \le t \le \pi$.

(a) Find the velocity vector $\vec{v}$ of the motion at $t = \pi/4$.

(b) Find acceleration vector $\vec{a}$ of the motion at $t = \pi/4$.

(c) Find the equation of the tangent line to the path of the motion at $t = \pi/4$.

10(15pts) For the force field $\vec{F}(x, y, z) = (y+z)\vec{i}+(-2x-z)\vec{j}+x\vec{k}$ and the line path C from $P(1, 0, -1)$ to $Q(2, 1, 1)$.

(a) Find a parameterized equation for C.

(b) Find the work done by $\vec{F}$ on an object that moves from P to Q on C.

11(15pts) Use the Divergence Theorem to set up an iterated integral for the flux of the vector field $\vec{F}(x, y, z) = xz\vec{i} + yx\vec{j} + zy\vec{k}$ through the surface of the rectangular solid $0 \le x \le 2, 0 \le y \le 1, 0 \le z \le 1$ oriented outward. **Do not evaluate the integral.**

12(15pts) Let S be the part of the parabola $z = f(x, y) = x^2 + y^2$ above the region: $x^2 + y^2 \le 1, x \ge 0, y \ge 0$, oriented upward.

(a) Find the differential area vector $d\vec{A}$ at any point of the surface.

(b) Set up an iterated double integral in polar coordinate for the flux of the vector field $\vec{F}(x, y, z) = y\vec{i} - x\vec{j} + z\vec{k}$ through S.

(c) Evaluate the integral from (b) to find the flux.