NAME:____

Circle which section you're in:

Burns	**Hamidi**	**Ismert**	**Kass**	**Ledder**	**Lindokken**
Packauskas	**Pei 10:30**	**Pei 12:30**	**Perez**	**Pollitz**	
Steinburg	**True 7:30**	**True 8:30**	**True 10:30**	**True 11:30**	

This exam should have 10 pages (including the title page); please check that it does. Show all work that you want considered for grading. Calculators are not allowed. **An answer will only be counted if it is supported by all the work necessary to get that answer.** Simplify as much as possible, except as noted: for example, don't write $\cos(\pi/4)$ when you can write $\sqrt{2}/2$. Also, **give exact answers only**, except as noted; for example, don't write 3.1415 for π. If there are multiple parts to a problem, the parts are not necessarily worth an equal amount of points.

Problem	**Maximum**	
1	12	
2	12	
3	12	
4	14	
5	14	
6	10	
7	12	
8	12	
9	14	
10	16	
11	16	
12	16	
13	14	
14	12	
15	14	
Total	200	

1. (12 points) The function $f(x, y) = x^4 - 4xy + 2y^2$ has three critical points: $(-1, -1)$, $(0, 0)$, and $(1, 1)$. Determine each one of them as a local max, local min, or a saddle point.

2. (12 points) Find all the critical points of $f(x, y) = x^2 + 14x - 2xy^2 + 4y^2 - 2$.
   DO NOT DO ANYTHING ELSE!

3. (12 points) An open-top rectangular box (no lid) is to be constructed from 12m^2 of cardboard. Use the method of Lagrange multipliers to set up a system of equations which can be used to find the maximum volume of such a box. You **do not** need to solve the system of equations.

4. (14 points) Let W be the region inside the sphere of radius 3 and above the cone $z = \sqrt{x^2 + y^2}$ with $y \geq 0$. Use an iterated integral in the most convenient coordinate system to compute the volume of W.

5. (14 points) Let $z = f(x, y) = x^2\sqrt{y} + 3$.

   (a) Find $f_x(2, 4)$.

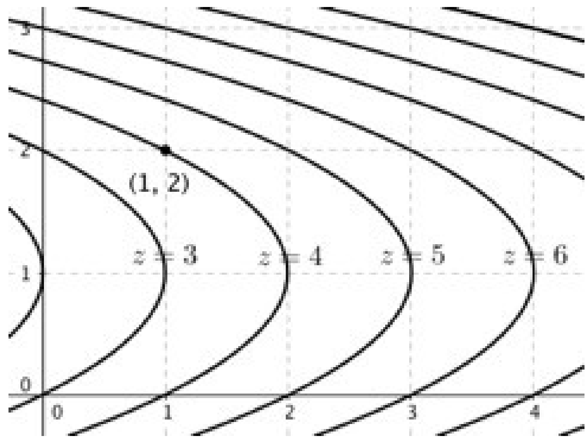
   (b) Find $f_y(2, 4)$.

   (c) Give the local linearization of $f(x, y)$ near (2,4).

   (d) Use the local linearization near (2,4) to approximate $f(2.02, 3.97)$.

   (e) Find the rate of change of f at $(2, 4)$ in the direction indicated by the vector $\vec{i} + \vec{j}$.

6. (10 points) The contour diagram for the function $z = f(x, y)$ is given below.



   (a) Estimate $f_x(1, 2)$. Show supporting work.

   (b) Is the value of $f_{xx}(1, 2)$ positive, negative or zero? Explain.

7. (12 points) Given the surface defined by $xy^3 + z^2 = 7$:

   (a) Find a vector orthogonal to the surface at (3, 1, 2)

   (b) Find an equation of the plane tangent to the surface at (3, 1, 2).

8. (12 points) Set up, **but do not evaluate**, an iterated integral that computes the mass of the solid in the first octant and below the plane $x + 2y + 3z = 6$, where the density at (x, y, z) is given by $\delta(x, y, z) = 6x$ g/cm^3.

9. (14 points) For the integral $\displaystyle\int_0^2 \int_{x/2}^1 \sin(y^2 + 1)\, dy\, dx$,

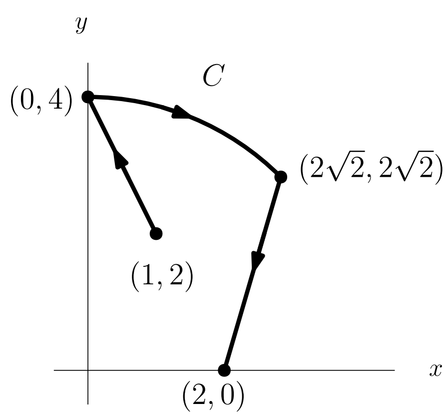
   (a) Sketch the integration region.

   (b) Evaluate the integral. (Hint: The function $\sin(y^2 + 1)$ does not have an elementary antiderivative.)

10. (16 points) Let $\vec{F} = 2xy\vec{i} + (x^2 + 8y^3)\vec{j}$

    (a) Find a potential function for $\vec{F}$

    (b) Calculate $\int_C \vec{F} \cdot d\vec{r}$ where C is the curve given by the line from $(1, 2)$ to $(0, 4)$, followed by the semicircle in the first quadrant that ends at the point $(2\sqrt{2}, 2\sqrt{2})$ and the line segment from that point to $(2, 0)$.



11. (16 points)

(a) Briefly explain what is wrong with the statement.

The gradient vector grad $f(x, y)$ points in the direction perpendicular to the surface $z = f(x, y)$.

(b) Briefly explain what is wrong with the statement.

If both the $x = a$ and $y = b$ cross sections of f are concave up at a critical point (a, b), then (a, b) must be a local minimum of f.

(c) Is the statement below true or false? Give a brief reason for your answer.

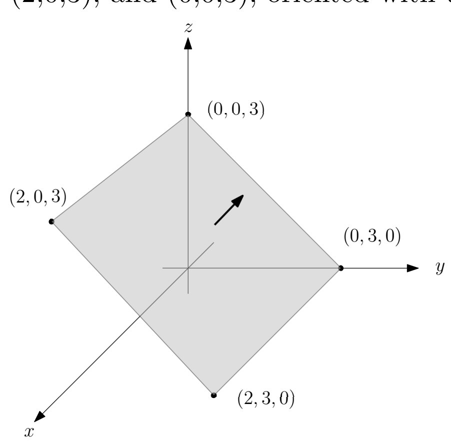
$$\int_0^1 \int_0^y xy\,dx\,dy = \int_0^y \int_0^1 xy\,dy\,dx$$

(d) Is the statement below true or false? Give a brief reason for your answer.

If $f(x)$ and $g(y)$ are smooth one-variable functions, then the vector field $\vec{F} = f(x)\vec{i} + g(y)\vec{j}$ is path-independent.

12. (16 points) Let C be the top half of the circle $x^2 + y^2 = 4$, oriented counterclockwise, and let $\vec{F}(x,y) = \dfrac{-y\vec{i} + x\vec{j}}{x^2+y^2}$. Use the parameterization method to calculate $\int_C \vec{F} \cdot d\vec{r}$.

13. (14 points) Set up, **but do not evaluate**, an iterated integral for the flux of $\vec{F}(x,y,z) = y\vec{i} - x\vec{j} + z^2\vec{k}$ through the portion of the plane $y + z = 3$ bounded by the rectangle with corners at (0,3,0), (2,3,0), (2,0,3), and (0,0,3), oriented with the normal pointing away from the origin.



14. (12 points) Set up, **but do not evaluate**, an iterated integral for the flux of the vector field

$$\vec{F}(x,y,z) = (z^2+x)\vec{i} + (y^2+x)\vec{j} - (y^2+z)\vec{k}$$

through the **closed** cylinder (top and bottom included) $x^2+y^2=4$ with $0 \le z \le 3$, oriented outward.

15. (14 points)

    (a) Calculate the curl of the vector field $\vec{F}(x,y,z) = 2z\vec{i} + y^2\vec{j} - x\vec{k}$.

    (b) Use Stokes' Theorem to compute $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F}$ is the vector field of part a and C is a circle of radius 6 in the plane $x+y+z=3$ centered at $(1,1,1)$, oriented counterclockwise when viewed from the origin.