MATH 208–FINAL EXAM

December 12, 2016

Name (Print):______________________________

Your Professor and Section (Circle Both):

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INSTRUCTIONS:

- *There are 8 pages of questions and this cover sheet.*
- *SHOW ALL YOUR WORK. Partial credit will be given only if your work is relevant and correct.*
- *Simplify your answers as much as possible.*
- *This examination is closed book. Calculators that perform symbolic manipulations such as the TI-89, TI-92 or their equivalents, are* ***not permitted****. Other calculators may be used. Turn off and put away all* ***smart watches, phones, and any devices capable of wireless communication.***

Question	Points	Score
1	16	
2	14	
3	12	
4	14	
5	14	
6	24	
7	20	
8	12	
9	12	
10	14	
11	12	
12	12	
13	12	
14	12	
Total	200	

1. [16 Points] Consider the vectors: $\overrightarrow{A} = 3\overrightarrow{i} + 2\overrightarrow{j} + \overrightarrow{k} = \langle 3, 2, 1 \rangle$, $\overrightarrow{B} = -2\overrightarrow{i} + \overrightarrow{k} = \langle -2, 0, 1 \rangle$.

a) [4 Points] Find $\cos\theta$, where $0 \leq \theta \leq \pi$ is the angle between between $\overrightarrow{A}$ and $\overrightarrow{B}$.

b) [4 Points] Compute $\overrightarrow{A} \times \overrightarrow{B}$.

c) [4 Points] Find the equation (in rectangular coordinates) of the plane $\mathcal{P}$ that contains the point $(2, -1, -1)$ and is parallel to the vectors $\overrightarrow{A}$ and $\overrightarrow{B}$.

d) [4 Points] Find the area of the parallelogram determined by the vectors $\overrightarrow{A}$ and $\overrightarrow{B}$.

2. [14 Points] **Find** and **classify** all critical points of the function: $f(x, y) = x^3 - 3xy - \frac{3}{2}y^2 + 6y$.

3. [12 Points] Find the equation of the tangent plane to the surface $\mathcal{S}: \ x^2 - y + z^2 - 6 = 0$ at the point $(2, -1, 1)$.

4. [14 Points] Use Lagrange multiplier to find the maximum and minimum values of $f(x, y, z) = x + 2y + 3z$; subject to the constraint $g(x, y, z) = x^2 + \frac{1}{2}y^2 + \frac{1}{2}z^2 - 108 = 0$.

5. [14 Points] Consider the function $f(x, y) = x^2 e^{y-1}$

a) [8 Points] Find $f_{\overrightarrow{u}}(2, 1)$, the directional derivative of f at $(2, 1)$ in the direction a unit vector $\overrightarrow{u}$, where $\overrightarrow{u}$ is in the direction of: $3\overrightarrow{i} - 4\overrightarrow{j} = \langle 3, -4 \rangle$.

b) [6 Points] Find the maximum rate of change of f at the point $(2, 1)$.

6. [24 Points] Let W be the solid that lies **above** the cone: $z = \sqrt{3}\,\sqrt{x^2 + y^2}$, and **under** the sphere: $x^2 + y^2 + z^2 = 16$. Express, **but don't evaluate**, the volume of W as:

a) [12 Points] an integral in **cylindrical coordinates**.

b) [12 Points] an integral in **spherical coordinates**.

7. [20 Points] Consider the vector field $\overrightarrow{F}(x, y) = 2xe^y \, \overrightarrow{i} + (3y^2 + x^2e^y) \, \overrightarrow{j} = \left\langle 2xe^y, \, 3y^2 + x^2e^y \right\rangle$.

a) [6 Points] Without finding a potential function, **carefully** show that $\overrightarrow{F}$ is a conservative vector field on $\mathbb{R}^2$, i.e., $\overrightarrow{F}$ is path-independent.

b) [8 Points] Find a potential function f so that $\overrightarrow{F}(x, y) = \nabla f(x, y)$.

c) [6 Points] Find $\displaystyle\int_{\mathcal{C}} \overrightarrow{F} \cdot d\overrightarrow{r}$, the work done by $\overrightarrow{F}$ in moving an object from $(1, 0)$ to $(3, 1)$ on any piecewise smooth plane curve $\mathcal{C}$.

8. [12 Points] Find $\displaystyle\int_{\mathcal{C}} \overrightarrow{F} \cdot d\overrightarrow{r}$, the work done by the force field $\overrightarrow{F}(x, y, z) = y \, \overrightarrow{i} + 3xy \, \overrightarrow{j} - 4z \, \overrightarrow{k} = \langle y, \, 3xy, \, -4z \rangle$ in moving an object on the **line segment** from $(1, 0, 0)$ to $(3, 4, 2)$ in $\mathbb{R}^3$.

9. [12 Points] **Sketch** the region of integration in the following integral, pass to polar coordinates and **evaluate** the resulting integral: $\displaystyle\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} e^{x^2+y^2}\,dydx.$

10. [14 Points] Let Ω be the solid region given by: $x^2+y^2 \le 1,\ 0 \le z \le 4$. Note that the boundary of Ω is the **closed** cylinder $\mathcal{S}$, oriented **outward**, where $\mathcal{S}$ consists of three surfaces:
$\mathcal{S}_1 : x^2+y^2 = 1,\ 0 \le z \le 4,$ $\quad\mathcal{S}_2 : x^2+y^2 \le 1,\ z=0,$ $\quad\mathcal{S}_3 : x^2+y^2 \le 1,\ z=4.$
Use the **divergence theorem only** to find $\displaystyle\int_{\mathcal{S}} \overrightarrow{F}\cdot d\overrightarrow{A}$, the flux of the vector field: $\overrightarrow{F}(x,y,z) = x^3\overrightarrow{i} + y^3\overrightarrow{j} + xy\overrightarrow{k} = \langle x^3, y^3, xy\rangle$ out of the closed surface $\mathcal{S}$. (Tip: You'll need to pass to cylindrical coordinates to evaluate the resulting integral).

11. [12 Points] **Sketch** the region of integration in the following integral, and **reverse** the order of integration. You **need not evaluate** the resulting integral: $\displaystyle\int_0^1 \int_{\sqrt{y}}^1 \cos(x^3)\, dx\, dy.$

12. [12 Points] Let $\mathcal{C}_1,\ \mathcal{C}_2$ be the curves given by:

$$\mathcal{C}_1 : x = \sqrt{4 - y^2},\ -2 \leq y \leq 2, \qquad \mathcal{C}_2 : x = 0,\ -2 \leq y \leq 2,$$

and $\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2$ is oriented counterclockwise. Let Ω be the region in $\mathbb{R}^2$ enclosed by $\mathcal{C}$ and $\overrightarrow{F}(x, y) = (-y + \cos^6 x)\,\overrightarrow{i} + 3x\,\overrightarrow{j} = \langle -y + \cos^6 x,\ 3x \rangle$. By using **Green's Theorem only**, find the value of the line integral: $\displaystyle\int_{\mathcal{C}} \overrightarrow{F} \cdot d\overrightarrow{r}$.

13. [12 Points] Let $\mathcal{S}$ be the portion of the paraboloid $z = 1 - x^2 - y^2$, $z \geq 0$, oriented upward. By directly evaluating the surface integral $\int_{\mathcal{S}} \overrightarrow{F} \cdot d\overrightarrow{A}$, find the flux of $\overrightarrow{F}$ through $\mathcal{S}$, where $\overrightarrow{F}(x, y, z) = x\,\overrightarrow{i} + y\,\overrightarrow{j} + z\,\overrightarrow{k} = \langle x,\, y,\, z \rangle$. (Hint: Pass to polar coordinates after setting up the surface integral).

14. [12 Points] Let $\mathcal{C}$ be the curve that consists of line segments from $P(1, 0, 0)$ to $Q(0, 1, 0)$, Q to $R(0, 0, 2)$, and from R back to P (note that $\mathcal{C}$ is the boundary of a triangle in the plane $2x + 2y + z - 2 = 0$). Use **Stokes' Theorem** to find, **but don't evaluate**, a double iterated integral in terms of x and y that gives the circulation of the vector field $\overrightarrow{F}(x, y, z) = yz\,\overrightarrow{i} - xz\,\overrightarrow{j} + z^2\,\overrightarrow{k} = \left\langle yz,\, -xz,\, z^2 \right\rangle$ around $\mathcal{C}$.