Math 107-Sec 510, Summer '04 **Exam 4** **Score:** ______

Name: ____________________ **TA's Name:** ____________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(16pts) (8pts each) (a) Find the Taylor polynomial $P_3(x)$ of $f(x) = (x+1)^{1/3}$ at point $x = 0$.

(b) Use Taylor's remainder formula

$$R_n(x) = f(x) - P_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1}, \text{ for some } z \text{ between } c \text{ and } x,$$

to estimate the number of digits that both $P_3(0.5)$ and $f(0.5)$ must agree.

(*Continue on Next Page ...*)

2(12pts) Find the interval on which the following power series converges absolutely. Also find the radius of convergence and discuss in details the convergence/divergence of the series at the end points of the interval of convergence.

$$\sum_{k=2}^{\infty} \frac{(x-1)^k}{k2^k}$$

3(16pts) (8pts each) (a) Use the Taylor series $\sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}, \quad -\infty < x < +\infty$ to find the Taylor series of

$$f(x) = \begin{cases} \frac{\sin 2x}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$$

and the interval of convergence.

(b) What is $f^{(72)}(0)$?

(*Continue on Next Page ...*)

4(16pts) (8pts each) (a) Using a known Taylor series to find the exact value of

$$\sum_{k=1}^{\infty}(-1)^k\frac{(1.5)^{2k}}{k!}$$

(b) For what n does the nth partial sum S_n approximate the series $S = \sum_{k=1}^{\infty}\frac{1}{k^{3/2}}$ to the first 3 decimal places? Show all the work. (*Hint*: Estimate the error by an integral.)

5(16pts) (8pts each) (a) Use the Taylor series for $\tan^{-1} x$ at $x = 0$ to represent the integral $\int_0^1 \frac{\tan^{-1} x}{x}$ as an infinite series. (If you don't remember the series, derive if from the series of $1/(1+x^2)$.)

(b) If we use the partial sum of the first 3 nonzero terms to approximate the integral, determine to which decimal place the approximation agrees with the integral. Explain your reasoning for the error estimate.

(*Continue on Next Page ...*)

6(24pts) (8pts each) Determine whether the following series converge absolutely, converge conditionally, or diverge. You must show all work to receive full credits, in particular the tests used.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt{k}+1}{2k^2+k+1}$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$

(c) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$

---

2 Bonus Points: Srinivasa Ramanujan is best known for his work in the area of: (a) algebra; (b) infinite series; (c) Euclidean geometry; (d) quantum mechanics. (... *The End*)