Math 107-Sec 510, Summer '04 **Exam 3** **Score:** _____

Name: _____ **TA's Name:** _____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(16pts) (8pts each) (a) Use the method of partial fraction to find the integral $\int \frac{3x+1}{x^2-4x+3}dx$.

(b) Find a correct form of partial fraction for $\frac{x^2-1}{(x^4+x^3)(x^2+6x+10)^2)}$. Do not solve for the constants.

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2(16pts) (8pts each) Determine by definition whether the improper integrals converge. Find the value of any convergent integral. **Make sure to show all details.**

(a) $\displaystyle\int_1^2 \frac{2}{\sqrt{x-1}}\,dx$

(b) $\displaystyle\int_0^\infty \frac{x}{1+x^2}\,dx$

3(12pts) Use the Comparison Test to determine whether or not the improper integral $\displaystyle\int_1^\infty \frac{x-\sqrt{x}}{1+x^3}dx$ converges. Verify all conditions carefully to apply the test.

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4(16pts) (8pts each) Determine whether the sequence converges. If it does, find the limit.

(a) $\lim_{n\to\infty} \dfrac{2\sqrt{n}+1}{n^2+n-1}$

(b) $\lim_{n\to\infty} \dfrac{(\ln n)^2}{\sqrt{n}}$ (*Hint:* Use L'Hopital Rule.)

5(8pts) Determine whether the sequence $a_n = \dfrac{n+2}{n+1}$ is monotone increasing or decreasing. Carefully show all the work.

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6(18pts) (9pts each) Determine if the series converge. Find the sum of any convergent series. **Make sure to include sufficient details.**

(a) $2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27} + \cdots + (-1)^k \frac{2^{k+1}}{3^k} + \cdots$

(b) $\sum_{k=1}^{\infty} \frac{2k+1}{k^2(k+1)^2}$. (*Hint*: Partial fraction the terms to apply telescoping cancellation for the partial sums.)

7(14pts) (7pts each) Determine whether the kth Term Test applies. If does, what is the conclusion?

(a) $\sum_{k=0}^{\infty} \frac{k}{2k+1}$

(b) $\sum_{k=1}^{\infty} (-1)^k \frac{1}{\sqrt{k}}$

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2 Bonus Points: The Pythagorean Theorem states: (... *The End*)