Math 107-Sec 510, Summer '04 **Exam 2** **Score:** _____

Name: _____ **TA's Name:** _____

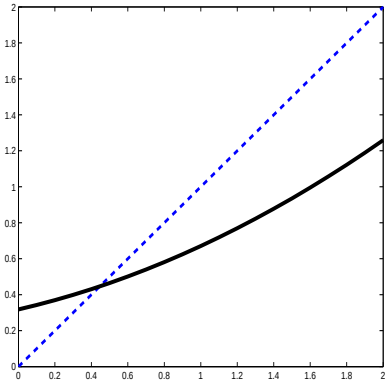
Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(20pts) Evaluate the integrals by the method of integration by parts.

(a) $\int xe^{2x}\,dx$

(b) $\int \sin^{-1} x\,dx$. (Note: $\dfrac{d}{dx}\sin^{-1} x = \dfrac{1}{\sqrt{1-x^2}}$.)

2(6pts) (c) An invertible function is as shown. Sketch the graph of its inverse.



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3(12pts) (a) Demonstrate that $y = f(x) = x^3 + x + 1$ is invertible.

(b) The tangent line to a function $y = f(x)$ at $(1, 2)$ is $y = 2 - 3(x - 1)$. Suppose f is invertible. Find the tangent line to the inverse function f^{-1} at the point $(2, 1)$.

4(16pts) (a) Derive and simplify $\frac{d}{dx} \csc^{-1} x$ for which $\csc^{-1} x$ is the inverse function of cosecant function $\csc x = \frac{1}{\sin x}$ from the interval of $(-\pi/2, 0) \cup (0, \pi/2)$ to $(-\infty, -1) \cup (1, \infty)$. Note that $\csc' x = -\csc x \cot x$.

(b) Scientists dating a fossil estimate that 20% of the original amount of carbon-14 is present. Given that the half-life is 5730 years, approximately how old is the fossil?

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5(16pts) Evaluate the integrals

(a) $\displaystyle\int \frac{3}{\sqrt{20-4x-2x^2}}\,dx$

(b) $\displaystyle\int_0^3 x^2\sqrt{x+1}\,dx$

6(10pts) Evaluate the integral $\displaystyle\int \tan^2\sec x dx$, using the following formulas

$$\int \sec^n x\,dx = \frac{1}{n-1}\sec^n x\tan x + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$$

$$\int \sec x\,dx = \ln|\sec x+\tan x| + C$$

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7(20pts) Evaluate the trigonometric integrals

(a) $\int \sin^4 x \, dx$

(b) $\int \frac{x^2}{\sqrt{x^2-4}} \, dx$, using trigonometric substitution.

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2 Bonus Points: My Math107 TA's new office will be room _____ of _____ Hall. (*... The End*)