Math 107-Sec 510, Summer '04 **Final Exam** **Score:** ____________

Name: ______________________________ **TA's Name:** ________________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

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score							

1(20pts) (5pts each)

(a) Find the right point sum R_1 for the function $y = x^2$ on the interval $[1, 2]$.

(b) Find the midpoint sum M_2 for the same function $y = x^2$ on the interval $[1, 2]$.

(c) If $R_{100} = 1.23$ and $L_{100} = 1.01$ for a function $y = f(x)$ over an interval $[a, b]$, what is the trapezoid sum T_{100}?

(d) For the same function f as above, if the midpoint sum $M_{100} = 1.11$, what is the Simpson sum S_{100}?

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2(18pts) (9pts each) (a) Find the Taylor polynomial $P_2(x)$ of $f(x) = x^{1/3}$ at point $x = 8$.

(b) To how many decimal places does $P_2(8.1)$ approximate the exact value of $8.1^{1/3}$? Explain your answer.

3(20pts) (10pts each) (a) Use the Taylor series for $\ln(1+x) = \sum_{k=1}^{\infty}(-1)^{k-1}x^k/k$ at $x = 0$ to represent the improper integral $\displaystyle\int_0^1 \frac{\ln(1+x^2)}{x}dx$ as an infinite series.

(b) If we use the partial sum of the first 3 nonzero terms to approximate the integral, determine to which decimal place the approximation agrees with the integral. Explain your reasoning for the error estimate.

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4(20pts) Find the interval on which the following power series converges absolutely. Also find the radius of convergence and discuss in details the convergence/divergence of the series at the end points of the interval of convergence.

$$\sum_{k=2}^{\infty} \frac{(x-1)^{k-1}}{2^k(\sqrt{k}-1)}$$

5(20pts) (10pts each) (a) Use the Taylor series $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$, $|x| < \infty$ to find the Taylor series of $\sinh(2x)$ at $c = 0$ where $\sinh x = \dfrac{e^x - e^{-x}}{2}$. What is the interval of convergence of the new series?

(b) Use the power rule $(x^r)' = rx^{r-1}$ and the Taylor series of $\dfrac{1}{1-x}$ at $c = 0$ to derive the Taylor series of $\dfrac{1}{(1-x)^2}$ at $c = 0$. What is the radius of convergence?

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6(20pts) (10pts each) Determine whether the following series converge absolutely, converge conditionally, or diverge. You must show all work to receive full credits.

(a) $\sum_{k=0}^{\infty} (-1)^k e^{-k}$

(b) $\sum_{k=2}^{\infty} \frac{(-1)^k}{k \ln k}$

7(16pts) (8pts each) Consider the parameterized curve $x(t) = t^3/3 - t, y(t) = t^2, \ -2 \leq t \leq 2$.

(a) Find all the points on the curve at which the tangent lines are vertical.

(b) Set up an integral for the area of the surface formed by revolving the curve about the x-axis. (**Do not evaluate the integral.**)

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8(18pts) (9pts each) (a) Find the sum of this finite series

$$\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\frac{1}{5\cdot 6}+\cdots+\frac{1}{21\cdot 22}$$

(b) Find the exact value of $\sum_{k=0}^{\infty}(-1)^k\frac{10^k}{11^{k+2}}$.

9(18pts) (6pts each) (a) Use your calculator to generate the graph of the polar equation $r = \sin 2\theta$. Copy the graph and find the corresponding xy-equation for the curve. (Hint: Use the double angle formula $\sin 2t = 2\sin t\cos t$.)

(a) Find the equation of the tangent line to the curve at the point corresponding to the parameter value $\theta = \pi/4$.

(b) Set up an integral for the length of one loop of the curve. Do not evaluate the integral.

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10(30pts) (10pts each) Find the integrals

(a) $\int \sin^2 x \cos^3 x dx$

(b) $\int_1^e x \ln x \, dx$ (Exact value please.)

(c) $\int \frac{1-x}{x^3+x} dx$

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2 Bonus Points: Who is considered the founding father of calculus? (a) Isaac Newton, (b) Johannes Kepler, (c) Gottfried Leibniz. (Circle all that apply.) (... *The End*)