Math 107-Sec 510, Summer '03 **Exam 3** **Score:** ____________

Name: ______________________________ **TA's Name:** ________________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(16pts) (8pts each) (a) Evaluate the integral $\int \frac{x^2+x+2}{x^3+2x^2}dx$

(b) Find a correct form of partial fraction for $\frac{2x^2+3x+3}{(x+1)^3(x^2+2x+3)^2}$. Do not solve for the constants.

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2(16pts) (8pts each) Determine by definition whether the improper integrals converge. Find the value of any convergent integral. **Make sure to show all details.**

(a) $\displaystyle\int_2^3 \frac{2x}{\sqrt{x^2-4}}\,dx$

(b) $\displaystyle\int_0^\infty \frac{\tan^{-1} x}{1+x^2}\,dx$

3(16pts) (8pts each) Use comparison tests to determine whether or not the improper integrals converge.

(a) $\displaystyle\int_1^\infty \frac{1}{1+x^3}dx$

(b) $\displaystyle\int_1^2 \frac{2+\sin x^2}{(x-1)^2}dx$

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4(18pts) (9pts each) Determine if the series converge. Find the sum of any convergent series. **Make sure to include sufficient details.**

(a) $\displaystyle\sum_{k=2}^{\infty}(-1)^k\frac{2^{k+1}}{3^k}$

(b) $\displaystyle\sum_{k=0}^{\infty}\frac{k\cos(1/k^2)}{2k+1}$

5(10pts) Use the Integral Test to determine if the series $\displaystyle\sum_{k=1}^{\infty} ke^{-k}$ converges. **Make sure to verify all the conditions of the test.**

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6(24pts) (8pts each) (a) Determine whether the sequence $\lim_{n\to\infty} \frac{(-1)^n n^2 + 1}{2n^2 + 2n + 1}$ converges. If it does, find the limit.

(b) Demonstrate that the sequence $a_n = \frac{n+2}{n+1}$ is monotone decreasing and find its limit $\lim_{n\to\infty} a_n$.

(c) Numerically approximate the infinite sum $\sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)2^k}$ to 4 decimal places. What is the minimum number of terms that is needed to obtain the required accuracy?

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2 Bonus Points: Pierre Simon Laplace worked on (a) a new calendar for Napoleon, (b) improper integrals to develop the Laplace transform, (c) a French vineyard as a slave laborer. (Circle all that are true)

(... *The End*)