Math 107-Sec 510, Summer '03 **Final Exam** **Score:** ____________

Name: ______________________________ **TA's Name:** ________________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

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score							

1(20pts) (10pts each) Determine whether the following series converge absolutely, converge conditionally, or diverge. You must show all work to receive full credits.

(a) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k k}{(\sqrt{k}+1)^2}$

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k (2k-1)}{k^{2.02}+1}$

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2(20pts) Find the interval on which the following power series converges absolutely. Also find the radius of convergence and discuss in details the convergence/divergence of the series at the end points of the interval of convergence.

$$\sum_{k=2}^{\infty} \frac{x^k}{k \ln k}$$

3(20pts) (10pts each) (a) Use the power series $\dfrac{1}{1-x} = \displaystyle\sum_{k=0}^{\infty} x^k$, $|x| < 1$ to find the Taylor series for $\dfrac{2x}{3+x^2}$, and the interval of convergence.

(b) Use the Taylor series of $\dfrac{2x}{3+x^2}$ from (a) to find the Taylor series of $\ln(3+x^2)$.

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4(18pts) (9pts each) (a) Find the Taylor polynomial $P_3(x)$ of $f(x) = \frac{1}{\sqrt{x}}$ at point $x = 1$.

(b) If we use the notation $(2k-1)!! = 1 \cdot 3 \cdot 5 \cdots (2k-1)$ and $(-1)!! := 1$, find the Taylor series for $f(x)$ at $x = 1$, and determine *ONLY the radius* of convergence.

5(20pts) (10pts each) (a) Use the Taylor series for $\sin x$ at $x = 0$ to represent the integral $\displaystyle\int_0^1 \frac{\sin x}{x}$ as an infinite series.

(b) If we use the partial sum of the first 3 nonzero terms to approximate the integral, determine to which decimal place the approximation agrees with the integral. Explain your reasoning for the error estimate.

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6(18pts) (9pts each) (a) Find the sum of this finite geometric series

$$\frac{2}{3}-\frac{1}{2}+\frac{3}{8}-\frac{9}{32}+\cdots-\frac{3^9 8}{2^{197}}$$

(b) Using a known Taylor series to find the exact value of

$$\sum_{k=1}^{\infty}(-1)^k\frac{10^k}{k!}$$

7(18pts) (9pts each) Consider the curve of 4-leaf rose $r=\sin 2\theta$

(a) Find the area of one leaf of the rose.

(b) Set up an integral for the arc length of one leaf of the rose, and approximate it numerically.

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8(30pts) (10pts each) Find the integrals

(a) $\displaystyle\int_{-1}^{1} x \sin \pi x \ dx$

(b) $\displaystyle\int \frac{2x-1}{x(x-1)} dx$

(c) $\displaystyle\int \sin^3 x dx$

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9(16pts) (8pts each) Consider the parameterized curve $x(t) = t^2, y(t) = t^2 + t,\ 0 \le t \le 1$.

(a) Find all the points on the curve at which the tangent lines are horizontal.

(b) Set up an integral for the area of the surface formed by revolving the curve about $x = -1$. (**Do not evaluate the integral.**)

10(20pts) (4pts each) The following table gives some values of a function $y = f(x)$:

x	1	1.1	1.2	1.3	1.4
f(x)	1	1.2	1.2	0.8	0.4

Approximate the value of the integral $\int_1^{1.4} f(x)dx$ by the following Riemann sums:

(a) The left point sum L_2

(b) The right point sum R_2

(c) The midpoint sum M_2

(d) The trapezoid sum T_2

(e) The Simpson sum S_2

2 Bonus Points: The Fundamental Theorem of Calculus is credited to (a) Isaac Newton, (b) Johannes Kepler, (c) Gottfried Leibniz. (Circle all that apply) (... *The End*)