Math 107-Sec 250 **Exam 3** **Name:** ____________________

Spring 2003 **Recitation Instructor:** ______________________

No.	1(a, b)	1(c)	2	3	4	5	6	7	Total
score									

1. (30 points, 10 points each) Determine whether the following series converge absolutely, converge conditionally or diverge. You must show all details to receive credit.

   a. $\displaystyle\sum_{k=1}^{\infty}(-1)^k \frac{3k^3-1}{k^{4.07}+2}$

   b. $\displaystyle\sum_{k=1}^{\infty}(-1)^k \left(\frac{4k-1}{3k+2}\right)^k$

c. $\sum_{k=1}^{\infty}(-1)^k \frac{99^k\, k!}{(2k+1)!}$

2. (18 points) Find the interval on which the following power series converges absolutely. Also, find the radius of convergence and make sure to **discuss in details** the convergence/divergence of the series at the end points of the interval you have found.

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{k} (x-2)^{k+1} \tag{1}$$

3. (9 points) Find the **first three non-zero terms** of the Taylor series of $f(x) = (1+x)^{1/3}$ about $x = 0$. You need not find every term in the series.

4. (8 points) By using the Taylor series: $\dfrac{1}{1-x} = \displaystyle\sum_{k=0}^{\infty} x^k$, $|x| < 1$; find the Taylor series of $f(x) = \dfrac{x^5}{9+x^2}$ about $x = 0$. Make sure to include the interval of convergence.

5. (8 points) By eliminating the parameter, **find** the xy-equation for the curve $\mathcal{C} : x = 2 - \cos t,\ \ y = -1 + \sin t,\ \ 0 \leq t \leq 2\pi$; and **sketch** $\mathcal{C}$ with orientation.

6. (12 points) Suppose that a function $f(x)$ has the following Taylor series about $x = 0$:

$$f(x) = \sum_{k=3}^{\infty} \frac{(-1)^k}{4^k(k+1)} x^{2k}, \quad -2 < x < 2. \tag{2}$$

a. (8 pts.) Find the exact value of $f^{(66)}(0)$.

b. (4 pts.) Find the Taylor series of $f'(x)$ about $x = 0$.

7. (15 points) Let $\mathcal{C}$ be a plane curve given by $\mathcal{C} : x = 1 + te^{t-1}, \; y = t^2 - 3t, \; 0 \leq t \leq 4$.

a. (10 pts.) Find the equation of the tangent line to $\mathcal{C}$ at the point that corresponds to $t = 1$.

b. (5 pts.) Find **but don't evaluate** a definite integral whose value gives the arc-length of $\mathcal{C}$.