Math 107-Sec 250 **Exam 1** **Name:** _____

Spring 2003 **Recitation Instructor:** _____

No.	1	2	3	4	5	6	Total
score							

1. (27 points, 9 points each) Evaluate each of the following integrals **(You must show all of your work to receive full credit. Here, no calculators allowed).**

   a. $\displaystyle\int \frac{\sin x}{3+\cos x}dx$

   b. $\displaystyle\int_1^e \frac{(\ln x)^7}{x}dx$

   c. $\displaystyle\int \frac{1}{\sqrt{x}}\sec^2\sqrt{x}\ dx$

2. (21 points, 7 points each) Let R be the region enclosed by $y = x^4$, $y = \sqrt{x}$ from $x = 0$ to $x = 1$.
a. Find **(but don't evaluate)** an integral whose value gives the exact area of the region R.

b. Find **(but don't evaluate)** an integral whose value gives the volume of the solid obtained by revolving the region R about the x-axis, **By using the method of slicing.**

c. Find **(but don't evaluate)** an integral whose value gives the volume of the solid obtained by revolving the region R about the y-axis, **By using the method of cylindrical shells.**

3. (21 points, 7 points each) A tank has a square base whose length is 5 feet and rectangular sides of height 3 feet. Assume that the tank is filled with water weighing $\rho = 62.5\ lb/ft^3$.

a. Find a Riemann sum whose value approximates the work required to pump all of the water over the top of the tank.

b. Write down **but do not evaluate** an integral whose value is exactly the work required to pump all of the water over the top of the tank.

c. Write down **but do not evaluate** an integral whose value is exactly the force exerted by the water on one side of the tank.

4. (8 points) Find $f^{-1}(x)$ if $y = f(x) = 4 + 3e^{2x}$.

5. (14 points) This question deals with the function $f(x) = 2x^3 + 5x - 1$.
a.(4 pts.) Show that f^{-1} exists.

b. (10 pts.) Find the equation of the tangent line to the function $y = f^{-1}(x)$ at the point $(6, 1)$.

6. (8 points) Let $y(t)$ be the amount of radioactive element present at time $t \geq 0$, and assume that $y(t)$ satisfies the equation: $\dfrac{dy}{dt} = -0.3y$. Write down the exact form of $y(t)$ and find the half life of the radioactive element.