Math 107, Ledder **EXAM 4: 4/10/03** **Name** ______________________

Show your work for full credit. **TA** ______________________

1. (a) (6 points) Use the geometric series $\sum_{k=0}^{\infty} x^k$ to derive power series representations for $\dfrac{1}{1+x^2}$ and $\dfrac{2x}{1+x^2}$.

   (b) (4 points) Use the results from a. to derive a power series representation for $\ln(1+x^2)$.

2. (a) (7 points) Use the integral test remainder estimates to determine upper and lower bounds for the error in using the approximation S_4 to approximate $\sum_{k=1}^{\infty} \dfrac{1}{k^4}$.

(b) (5 points) Compute $\sum_{k=1}^{\infty} \frac{4}{3^k}$ or show that the series diverges.

3. The Taylor series representation for $f(x)$ near $x = 2$ is $\sum_{k=0}^{\infty} (-1)^k \frac{k}{5^k} (x-2)^k$.

(a) (8 points) Find the radius of convergence of the series.

(b) (3 points) Find the interval of convergence of the series (ignore the endpoints).

(c) (5 points) Use the series to obtain the best quadratic (2nd degree) approximation for $f(2.1)$. (Report the exact answer.)

4. Let $f(x) = \ln(2 + x)$.

(a) (10 points) Use the Taylor coefficient formula to find the Taylor polynomial (centered at 0) $\mathrm{P}_3(x)$ for f.

(b) (6 points) Determine an upper bound for the remainder $\mathrm{R}_3(1)$ in the use of $\mathrm{P}_3(x)$ to approximate $f(1)$.

5. (6 points) Determine whether or not the series $\displaystyle\sum_{k=0}^{\infty} \frac{(k!)^2}{(2k)!}$ converges.